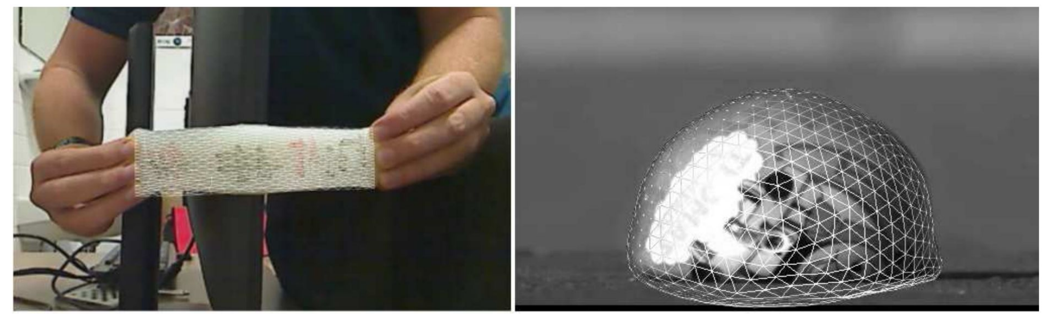

Template-based Monocular 3D Recovery of Elastic Shapes using Lagrangian Multipliers

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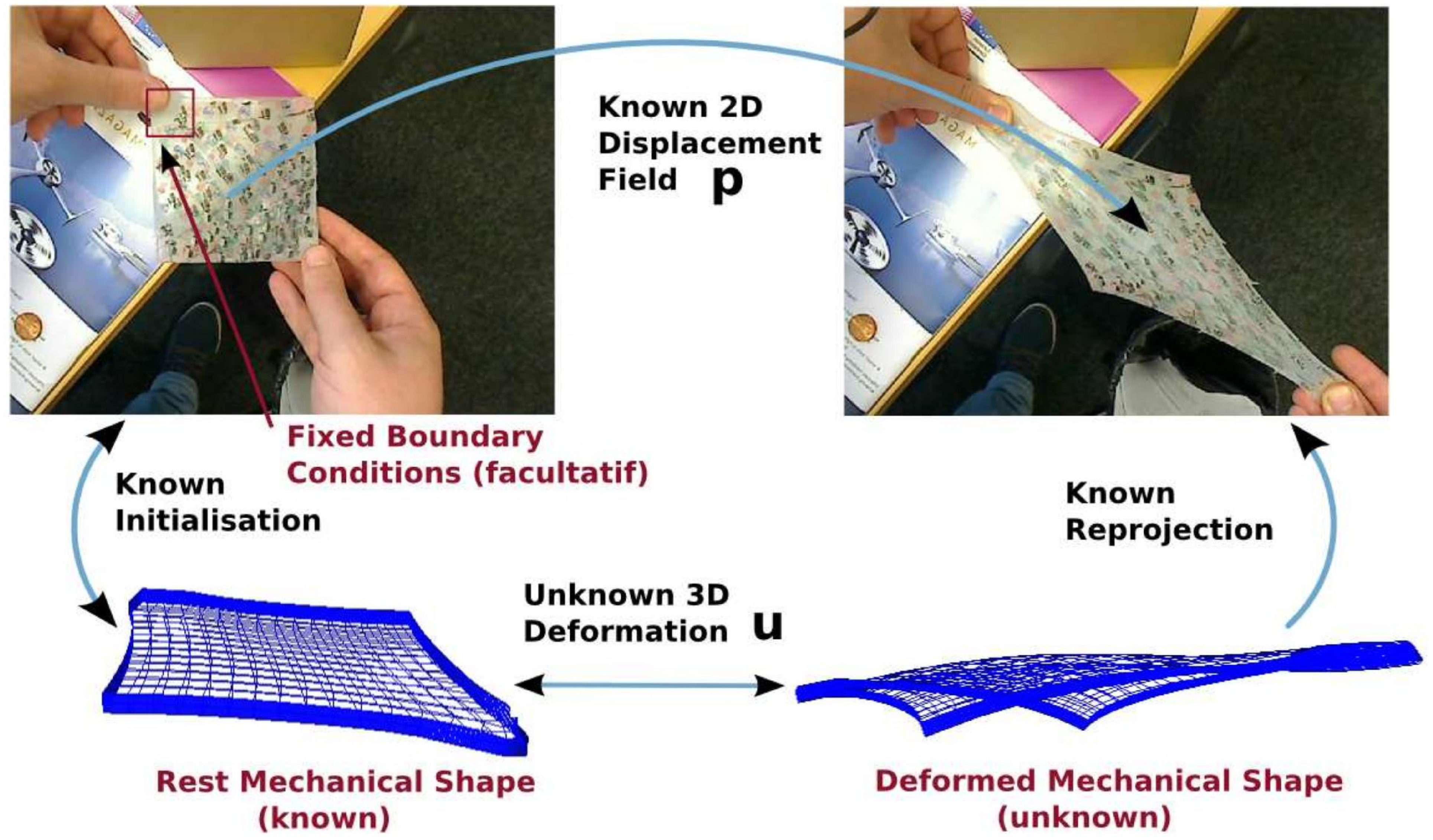
ELASTIC SHAPE-FROM-TEMPLATE:

- Extensible surface, monocular camera
- Underconstrained ⇒ several shapes produce the same projection
- No preservation of mesh geometrical properties, no temporal consistency and no shading information



Contribution:

- truly handles large elastic deformations up to 130 %
- is invariant to material properties
- is adapted to both extensible and inextensible materials

PROBLEM FORMULATION:

Find 3D point **u**, from 2D point **p**, that satisfies physical constraints **K (stiffness matrix)** and camera constraints **L (projection matrix)** with **f** being the force

$$\begin{cases} \mathbf{Ku} &= \mathbf{f} \\ \mathbf{Lu} &= \mathbf{p} \end{cases}$$

PHYSICS-BASED MODEL:

- Saint-Venant Kirchhoff material
- Material parameters are Young's modulus (stiffness) and Poisson's ratio (compressibility)
- Volume built from the mesh (template)
- Finite Element Method for discretization
- Tetrahedral elements
- Static scheme: no acceleration, no mass, no damping
- 2D points **p** are considered as BCs (fixed or moving)
- SOFA framework [1]

LAGRANGIAN MULTIPLIERS:

- Constrained minimization problem

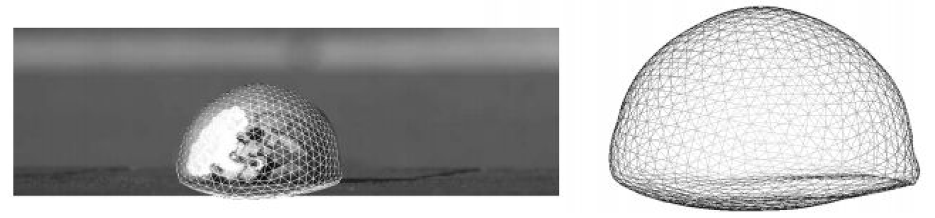
$$\min_{\mathbf{u}\in\mathbb{R}^{3n}} \{\frac{1}{2}\mathbf{u}^\top\mathbf{K}\mathbf{u} - \mathbf{u}^\top\mathbf{f} : \mathbf{Lu} = \mathbf{p}\}$$

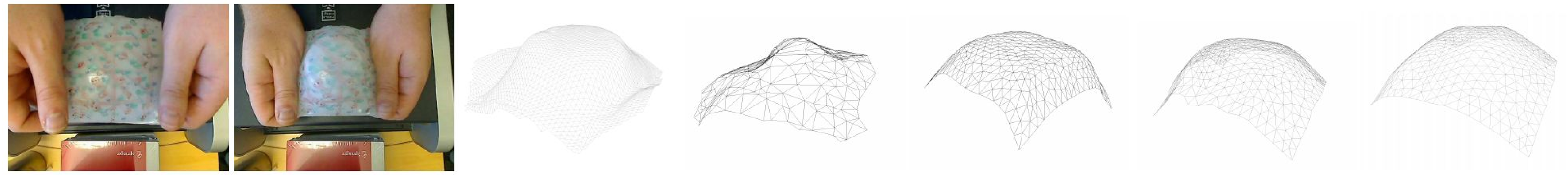
- Expressed as a saddle point problem
- Uses of Lagrangian multipliers method by adjoining the vector $\boldsymbol{\lambda}$ to the system of equation
- Results in a linear system

$$\begin{bmatrix} \mathbf{K} & \mathbf{L}^\top \\ \mathbf{L} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{u} \\ \boldsymbol{\lambda} \end{bmatrix} = \begin{bmatrix} \mathbf{f} \\ \mathbf{p} \end{bmatrix}$$

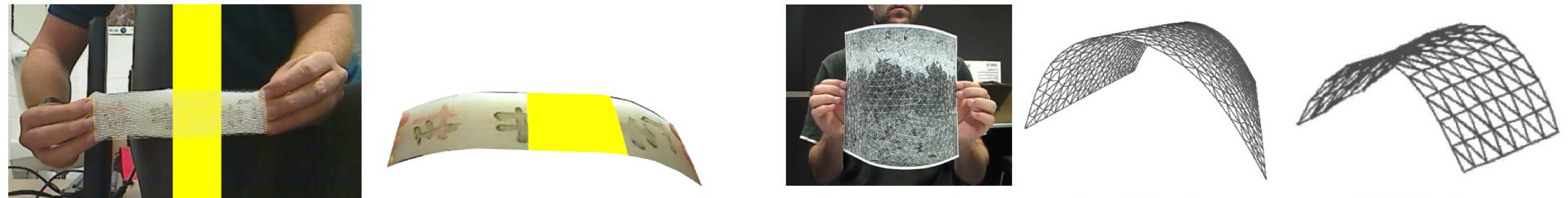
RESULTS ON REAL DATA:

- Stretching, torsion, compression ...
- Poorly-textured surfaces and partially occluded areas
- No preliminary knowledge of material properties



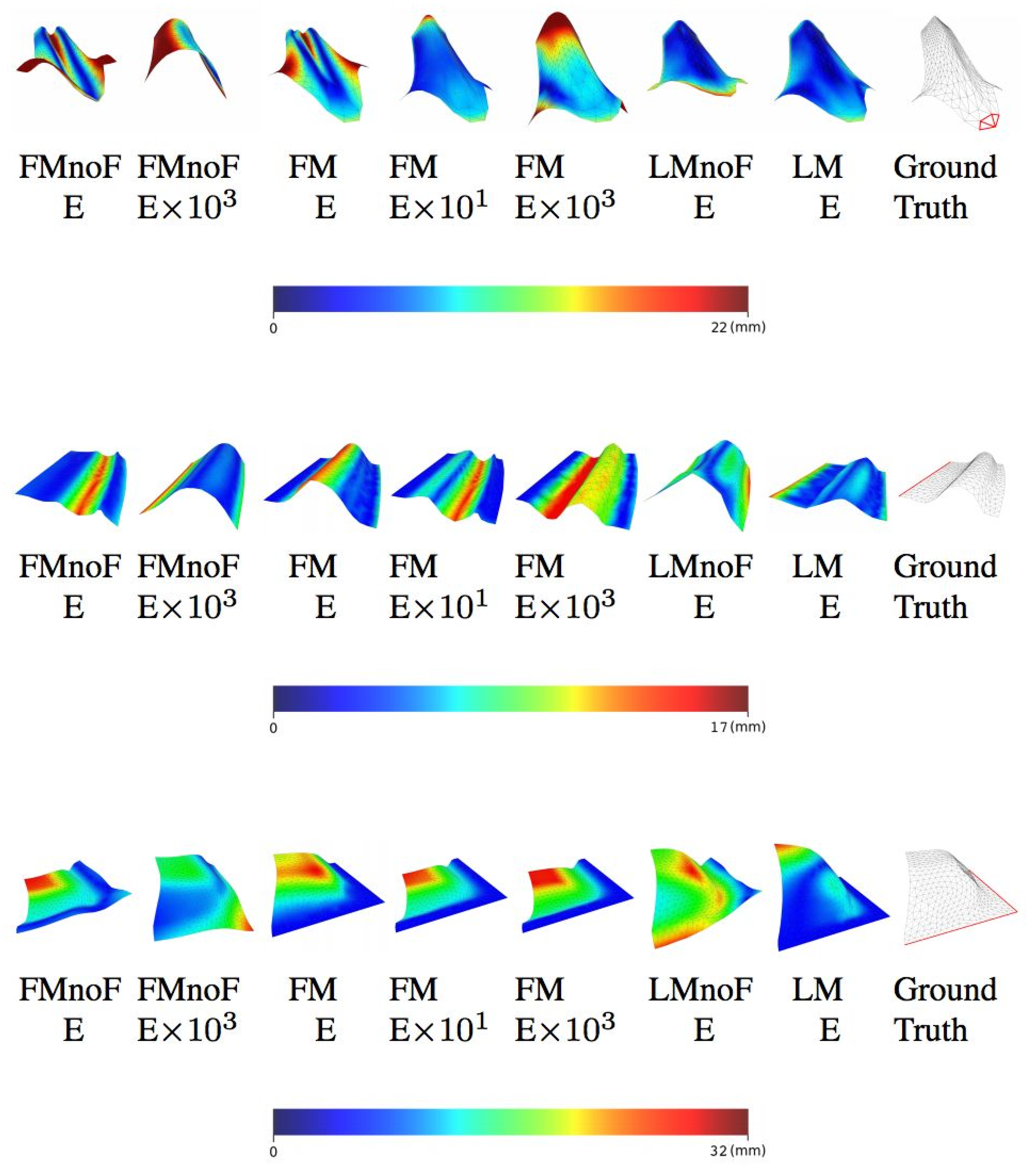


Rest state | Deformed state | Ground Truth | Inext | LLS | FM | LM



3D mesh overlay | Our **LM** method | **CVX** method

EVALUATION:



- Results on synthetic data with variation of Young's Modulus E with and without fixed boundary conditions (red in the ground truth).
- Our method LM is able to recover the 3D shape and is invariant to material properties.

FUTURE DIRECTION:

Mismatched points ⇒ Elastic invariant feature descriptor (modal analysis)

[1] www.sofa-framework.org