On the Two-View Geometry of Unsynchronized Cameras

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WHAT?

- Minimal solvers for two-view geometry with a time shift
- Iterative algorithm for robust space-time camera calibration
- Synchronizes sequences several seconds apart
- Achieves sub-frame synchronization

WHY?

- Multi-camera systems perform 3D reconstruction, tracking, localization etc.
- They require precise time and space calibration

Existing methods

- Require special content
- Sensitive to outliers and initialization
- Exhaustive and time-consuming

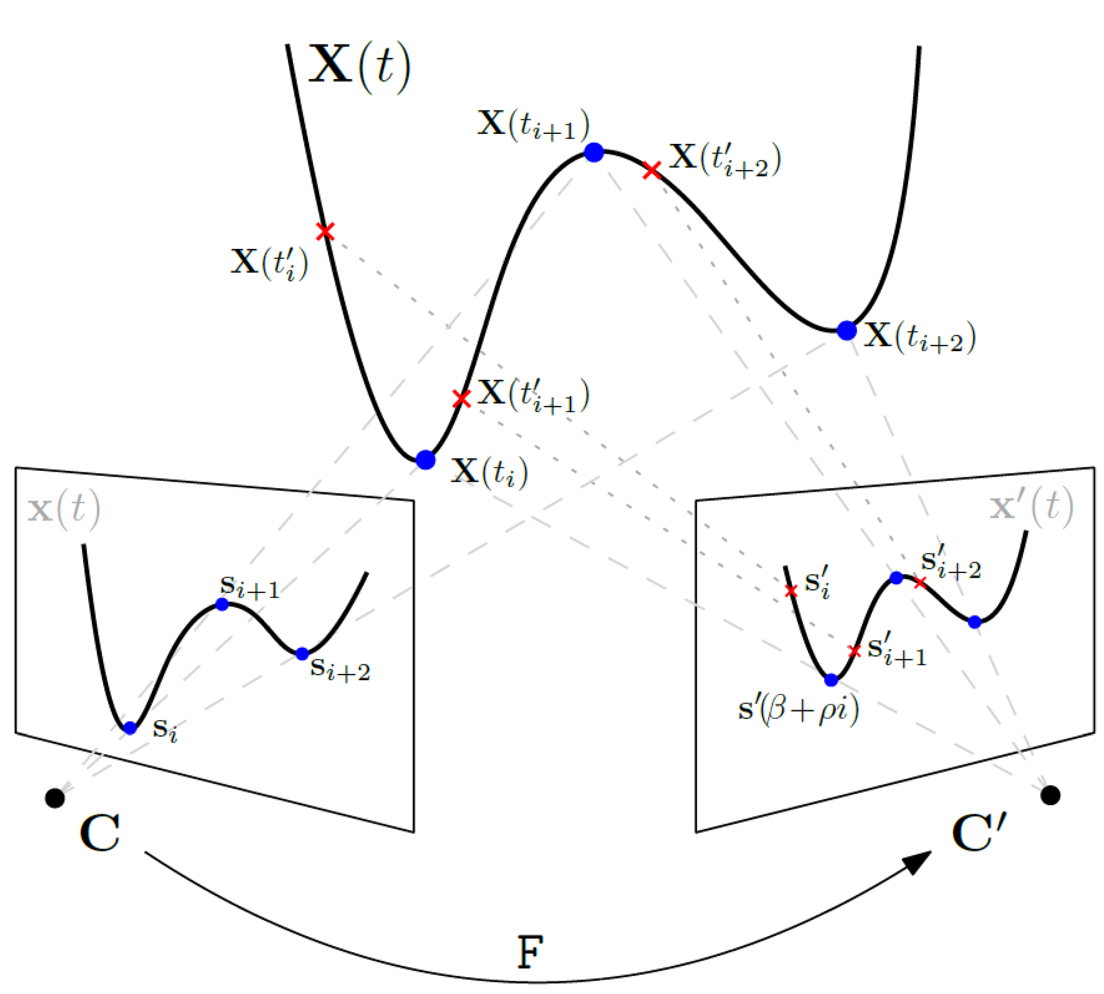
Our method

- Works with general data
- Robust
- Efficient

HOW?

Image trajectory approximation

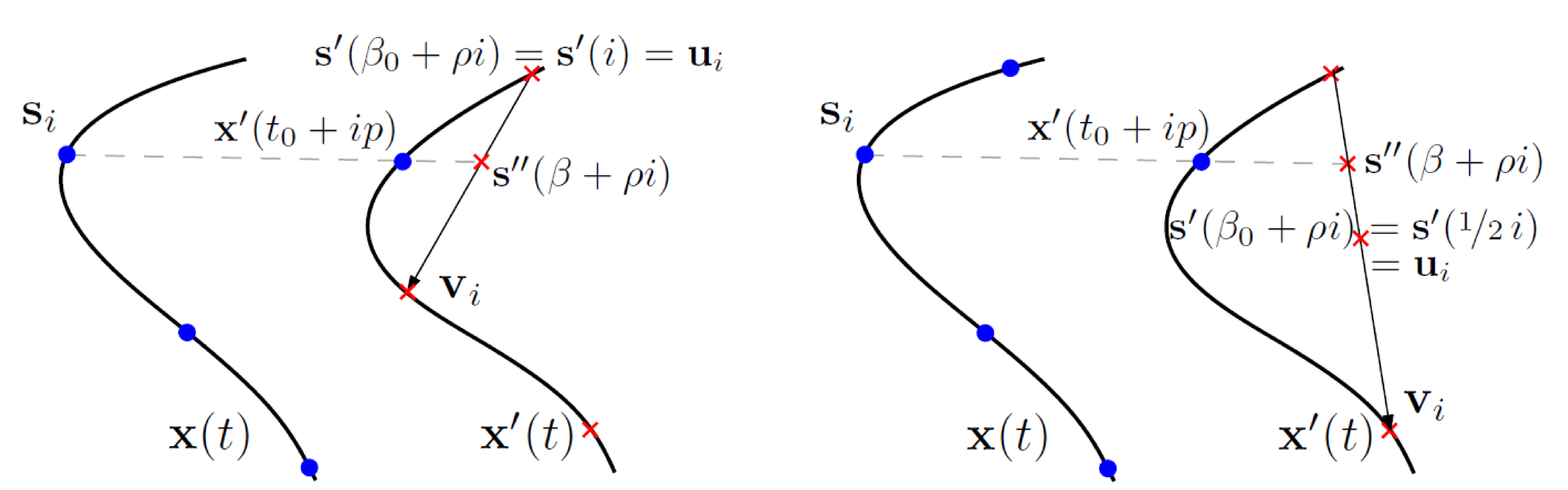
- Cameras (in a rig) sample a moving object with trajectory $\mathbf{X}(t'_j)$ (or moving camera rig+static object) as samples $\mathbf{s}_i$ and $\mathbf{s}'_j$



- The object trajectory in the second image $\mathbf{x}'(t'_j)$ is approximated by a function such that $\mathbf{s}_i \longleftrightarrow \mathbf{s}'(\beta + \rho i)$

Linearized trajectory

- To solve for F or H and time shift β we assume linear trajectory



$$\mathbf{s}'(\beta + \rho i) \approx \mathbf{s}'(\beta_0 + \rho i) + (\beta - \beta_0)\mathbf{v} = \mathbf{s}''(\beta + \rho i)$$

$$\mathbf{v} = \mathbf{s}'_{j_0+d} - \mathbf{s}'_{j_0}$$

Epipolar geometry

$$\mathbf{s}'(\beta + \rho i)^\top \mathbf{F}\mathbf{s}_i = 0 \Longrightarrow (\mathbf{u}_i + \beta\mathbf{v}_i)^\top \mathbf{F}\mathbf{s}_i = 0$$

Homography

$$\mathbf{H}\mathbf{s}_i = \lambda_i (\mathbf{u}_i + \beta\mathbf{v}_i)$$

MINIMAL SOLUTIONS

Epipolar geometry

- $(\mathbf{u}_i + \beta\mathbf{v}_i)^\top \mathbf{F}\mathbf{s}_i = 0$ can be written as $\mathtt{M}\mathbf{w} = \mathbf{0}$

$$\mathbf{w} = [f_{11}, f_{12}, f_{13}, f_{21}, f_{22}, f_{23}, f_{31}, f_{32}, f_{33}, \beta f_{11}, \beta f_{12}, \beta f_{13}, \beta f_{21}, \beta f_{22}, \beta f_{23}]$$

- The minimal number of samples - 8
- $\mathtt{M}$ is a 8x15 matrix of rank 8
- F defined up to scale, write $\mathbf{w}$ using the null space of $\mathtt{M}$

$$\mathbf{w} = \mathbf{n}_0 + \sum_{i=1}^{6} \alpha_i \mathbf{n}_i$$

- Additionally the elements of $\mathbf{w}$ satisfy

$$\beta w_j = w_k \quad \text{for } (j,k) \in \{(1,10), ..., (6,15)\}$$

- Quite complicated system of 7 polynomial equations in 7 unknowns (solver size 633x649)
- SOLUTION – eliminate β first using elimination ideal method [1] and then solve by automatic generator [2]
- Solver size 194x210 with 16 solutions

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Homography

- $[\mathbf{u}_i + \beta \mathbf{v}_i]_\times \mathtt{H} \mathbf{s}_i = \mathbf{0}$ – 3 equations – 2 linearly independent
- The minimal number of samples – 4.5 (5)
- Equations corresponding to the first and second row in a matrix form

$$\mathtt{M}\,\mathbf{w} = \mathbf{0}$$

$$\mathbf{w} = [h_{11}, h_{12}, h_{13}, h_{21}, h_{22}, h_{23}, h_{31}, h_{32}, h_{33}, \beta\, h_{31}, \beta\, h_{32}, \beta\, h_{33}]^\top$$

- $\mathtt{M}$ is a 9x12 matrix of rank 9 ⟹ 3dim null space

$$\mathbf{w} = \sum_{i=1}^{3} \gamma_i \mathbf{n}_i$$

- H is defined up to scale ⟹ we can fix $\gamma_3 = 1$
- The elements of $\mathbf{w}$ satisfy

$$w_{10} = \beta\, w_7, w_{11} = \beta\, w_8, w_{12} = \beta\, w_9$$

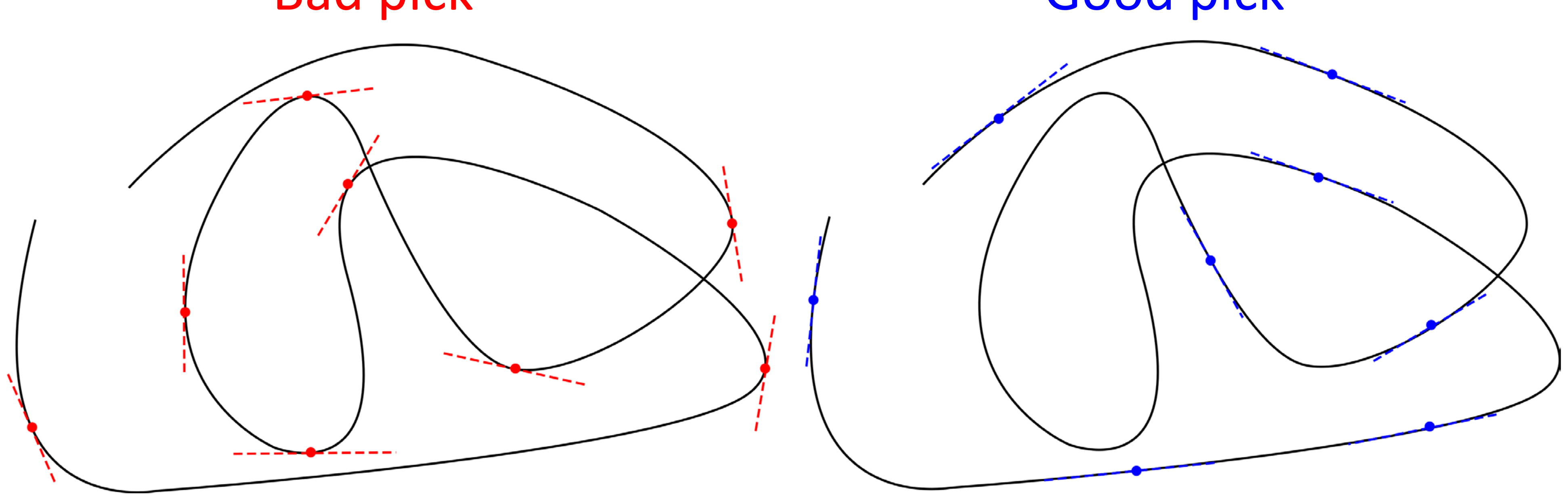
- System of 3 quadratic equations in 3 unknowns and 6 monomials
- Solved by G-J elimination of the 3x6 coefficient matrix and computing eigenvalues of a 3x3 matrix

USING RANSAC

- RANSAC use has 2 reasons:
  - Outliers in the data, mismatches etc.
  - Outliers due to the linearization of the trajectory

Bad pick

Good pick



ITERATIVE ALGORITHM

- For larger time shifts (multiple frames)
- When time shift is large, increasing interpolation distance (e.g. next 2^k-th sample) improves the estimate of β
- Start with interpolation distance d = 1 frame
- **LOOP**

  RANSAC F or H and β

  IF no improvement in inliers

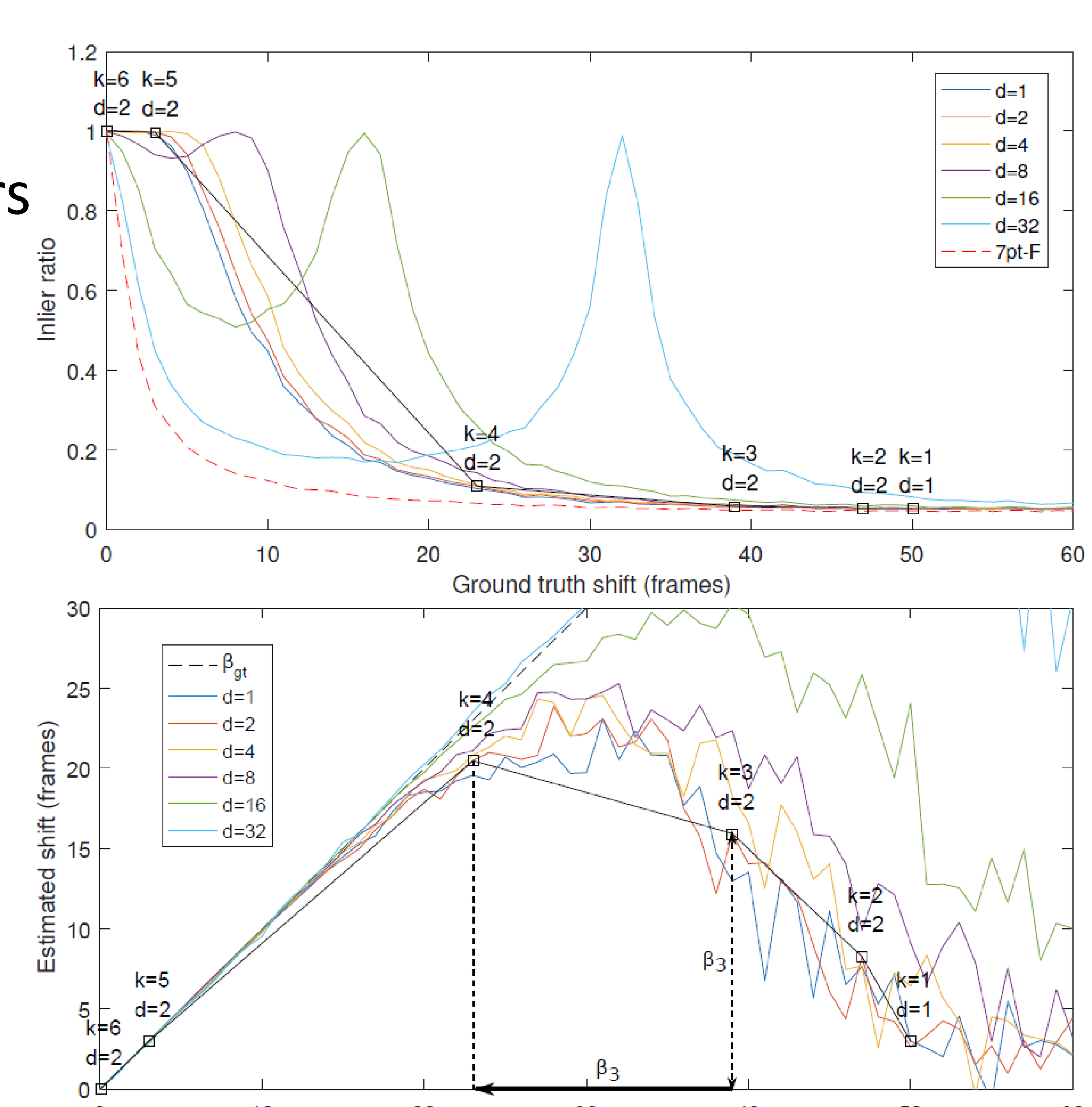
  increase d (e.g. d = 2^k)

  ELSE

  Correct the shift by β

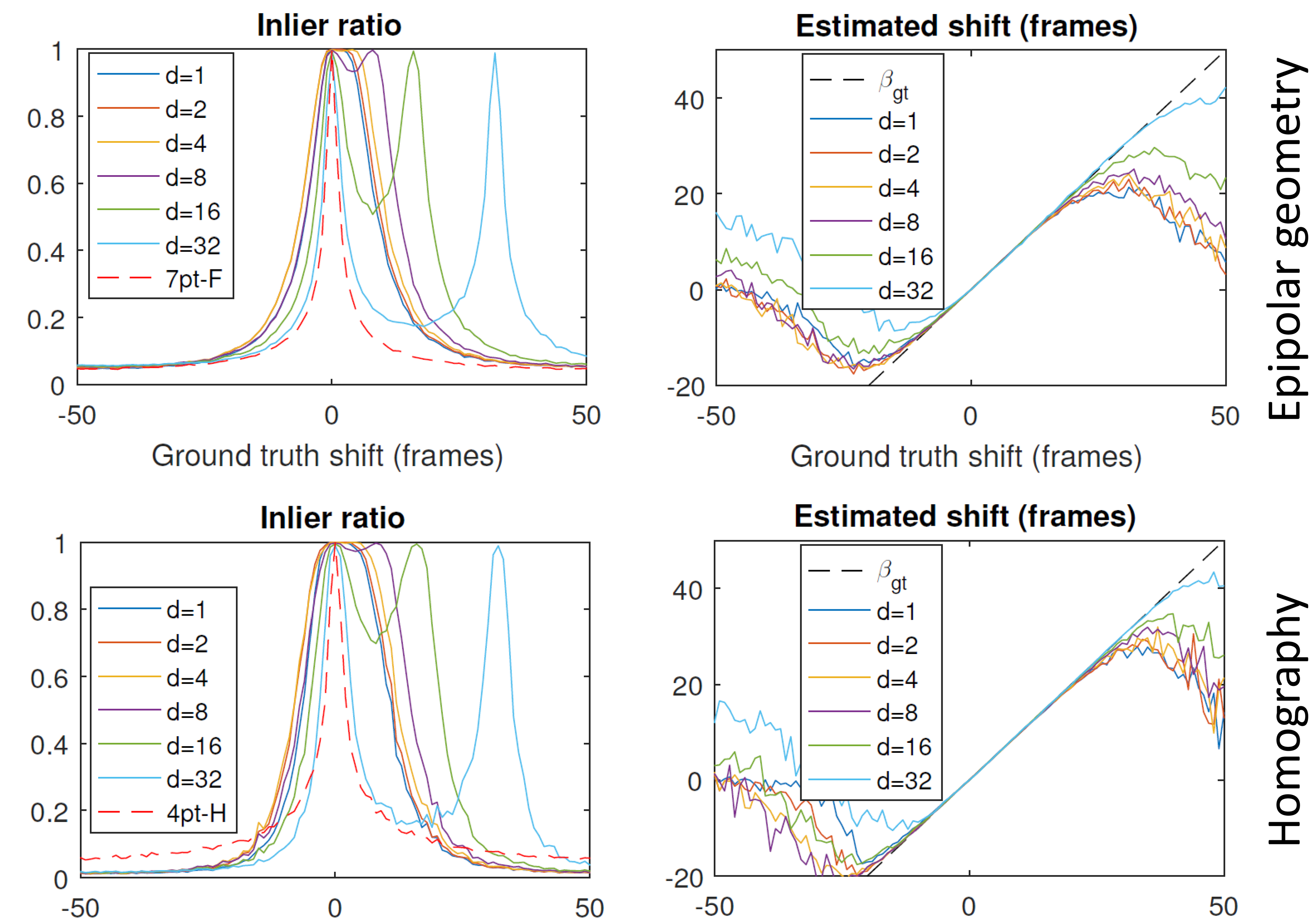
  END IF

  END LOOP

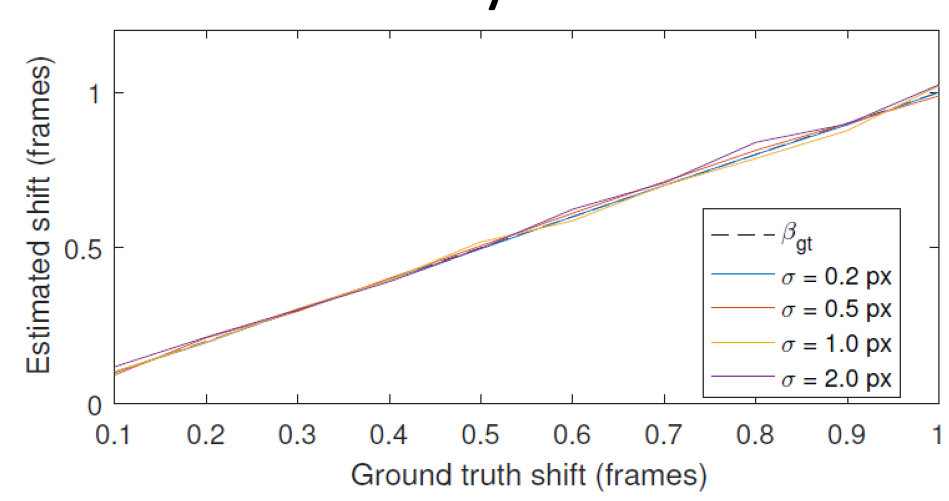


- Algorithm stops when all interpolation distances up to 2^{kmax} have failed to provided more inliers at current time shift estimate

RESULTS

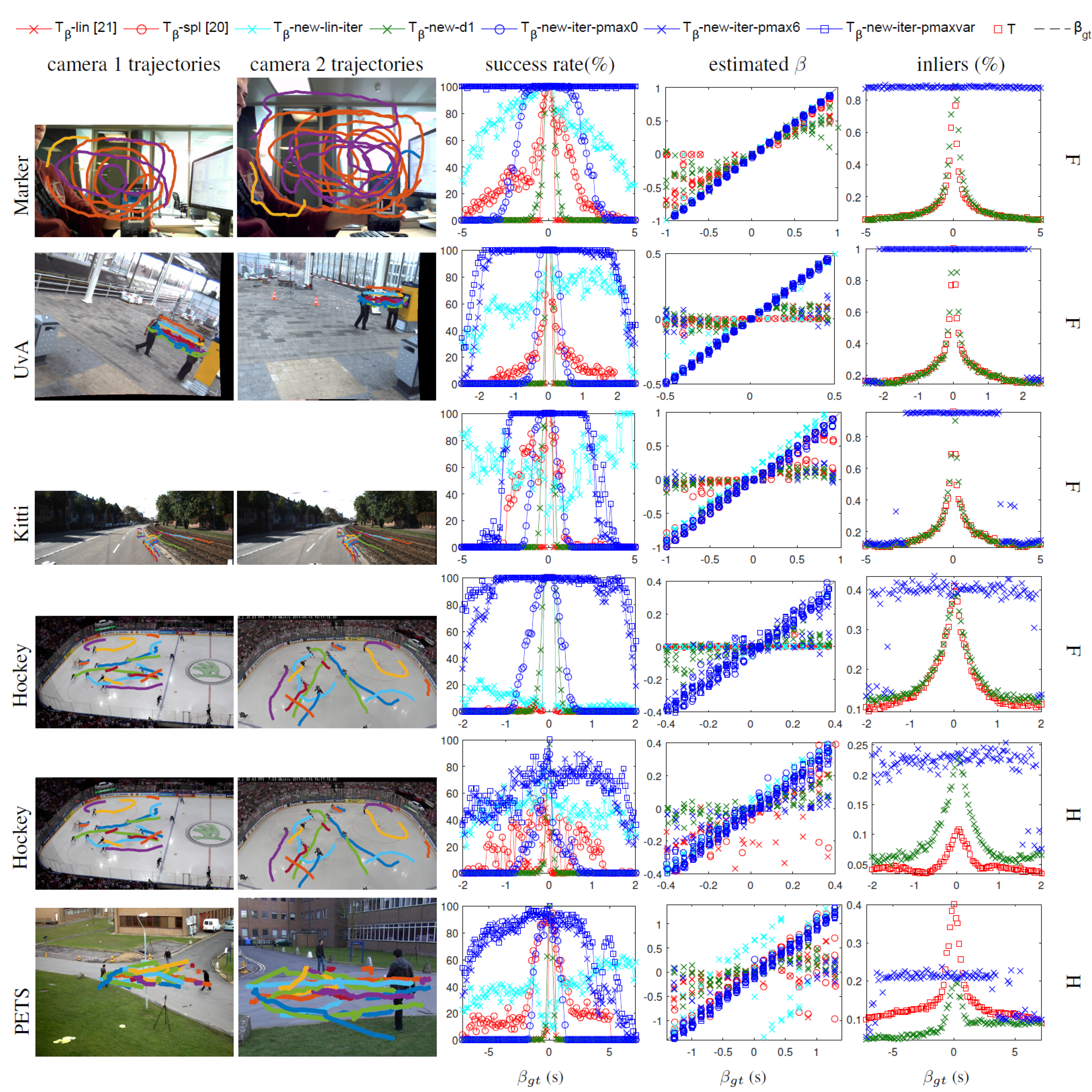


Sub-frame synchronization



Typical number of iterations for various time shifts (in frames)

β_{gt}	0-10	10-20	20-30	30-40	40-50
T_β-new-iter-pmax0	4.7	4.3	3.5	4.1	3.8
T_β-new-iter-pmax6	23	22	21.2	21.6	21.2
T_β-new-iter-pmaxvar	18	19	17.5	16.7	16.5



REFERENCES

[1] Z. Kukelova, J. Kileel, B. Sturmfels, and T. Pajdla. A clever elimination strategy for efficient minimal solvers. CVPR, 2017.

[2] Z. Kukelova, M. Bujnak, and T. Pajdla. Automatic generator of minimal problem solvers. ECCV, 2008.