Convex Global 3D Registration with Lagrangian Duality

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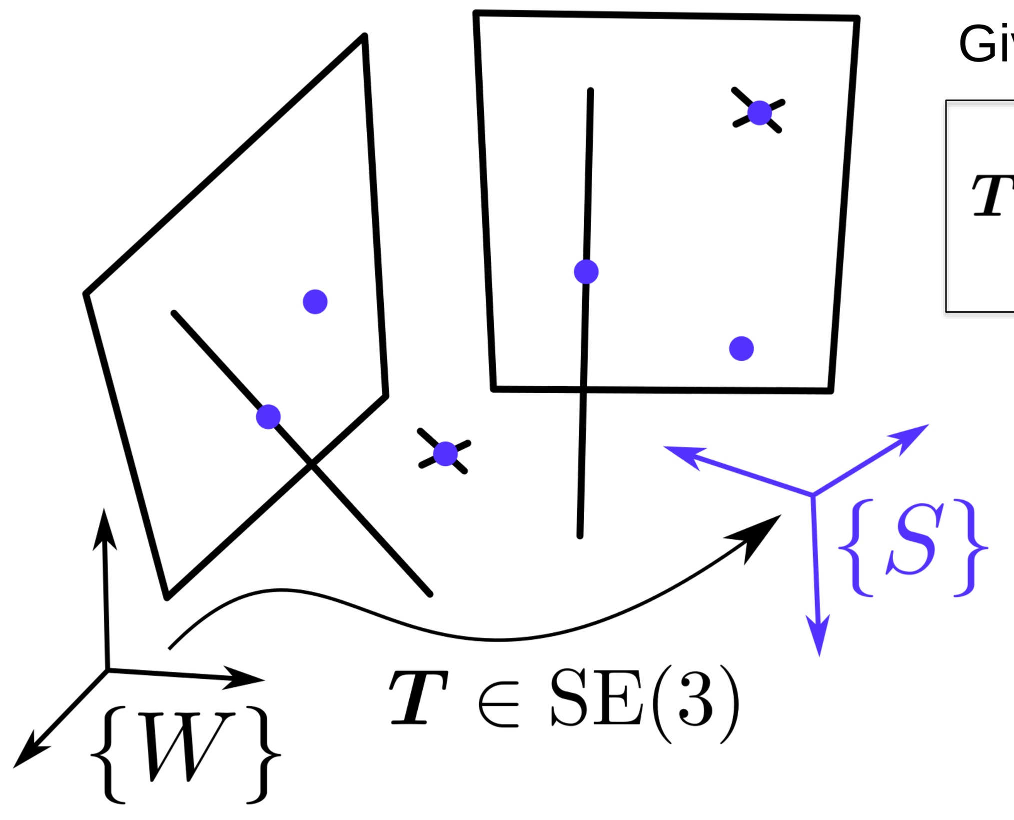
The problem: Rigid registration of *points*, *lines* and *planes*



Given point-to-any correspondences:

$$T^\star = \underset{T\in SE(3)}{\arg\min} \sum_{i=1}^{m} \underbrace{\min_{y_i\in P_i} \|T \oplus x_i - y_i\|_2^2}_{\mathrm{d}_{P_i}(T\oplus x_i)^2}$$

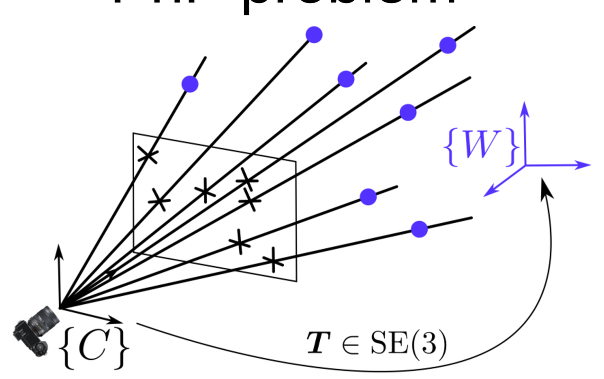
Unknowns:

$$T = [R|t] \in SE(3) \equiv SO(3)\times\mathbb{R}^3$$

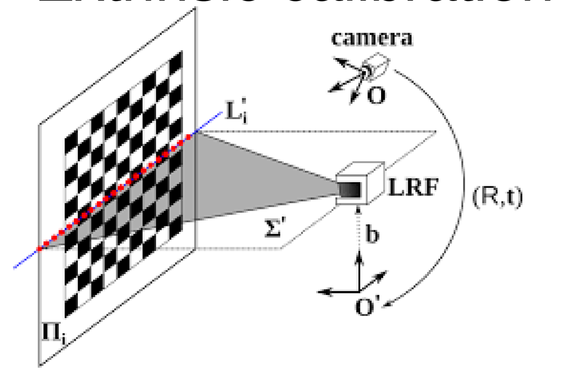
$$R = [c_1|c_2|c_3] = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix}$$

Examples: *Lines* and *planes* are pervasive in Computer Vision

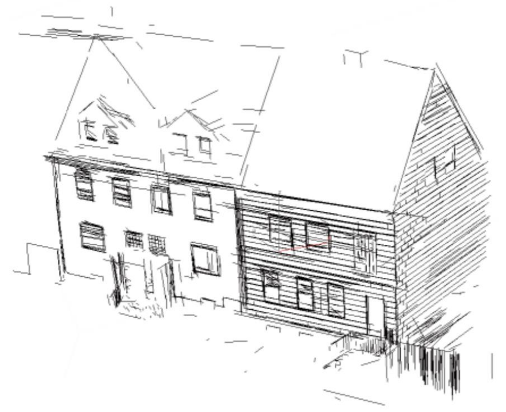
PnP problem



Extrinsic calibration



Rich models



Goal: Find *globally* optimal pose, *fast*

- Alternatives are either slow (BnB [1]) or suboptimal (SDP [2])

Equivalent formulation: Quadratic objective

- Generalized distance: $\mathrm{d}_P(x)^2 = \|x - y\|_C^2$
- Linear transformation: $T \oplus x_i = X_i \,\mathrm{vec}(T)$
- Marginalization of translation: $t \in \mathbb{R}^3$

Quadratic objective: $f(r) = \tilde{r}^\top \tilde{Q} \tilde{r}$

$$f^\star = \min_{R\in SO(3)} \tilde{r}^\top \tilde{Q}\tilde{r}, \quad \tilde{r} = \begin{bmatrix} r \\ 1 \end{bmatrix}, \quad r = \mathrm{vec}(R)$$

Rotation constraint:
$R \in SO(3) \Leftrightarrow \{c_i(r) = 0\}$

Non-convex!

How to solve this non-convex problem globally?

Convex relaxation via Lagrangian duality

($\mathcal{P}$) Primal problem: QCQP

Equivalent to original problem
Non-convex optimization

$$f^\star = \min_r f(r) \quad \text{s.t. } c_i(r) = 0$$

$$f(r) = \tilde{r}^\top \tilde{Q}\tilde{r}, \; c_i(r) = \tilde{r}^\top \tilde{P}_i \tilde{r}$$

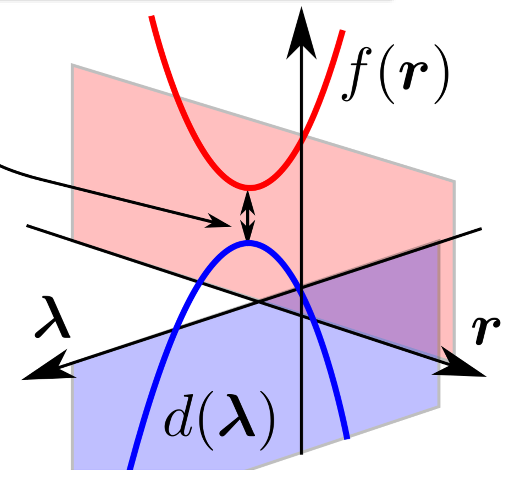
Quadratic objective and constraints

($\mathcal{R}$) Lagrangian relaxation

Subs. constraints by penalizations

$$\mathrm{L}(\tilde{r}, \tilde{\lambda}) = f(r) + \sum \lambda_i c_i(r)$$
$$= \tilde{r}^\top (\tilde{Q} + \sum \lambda_i \tilde{P}_i)\tilde{r}$$
$$d(\lambda) = \min_r \mathrm{L}(r, \lambda) \le f^\star$$

$d(\lambda^\star) \le f(r^\star)$
(*weak duality*)



If $d^\star = f^\star$ holds
(*strong duality*)

($\mathcal{D}$) Dual problem: SDP

Find best relaxation
Convex optimization

$$d^\star = \max_\lambda d(\lambda) \ge d(\lambda)$$

$$d^\star = \max_{\tilde{\lambda}} \gamma, \;\text{s.t. } \tilde{Q} + \sum_i \lambda_i \tilde{P}_i \succcurlyeq 0$$

Linear objective
LMI constraint

$$\tilde{\lambda} = \begin{bmatrix} \lambda \\ \gamma \end{bmatrix}$$

Primal-via-dual recovery

$$r^\star = \arg\min_r \mathrm{L}(r, \lambda^\star)$$
$$\tilde{r}^\star \in \mathrm{null}(\tilde{Q} + \sum \lambda_i^\star \tilde{P}_i)$$

Main contribution: Exploit redundant rotation constraints

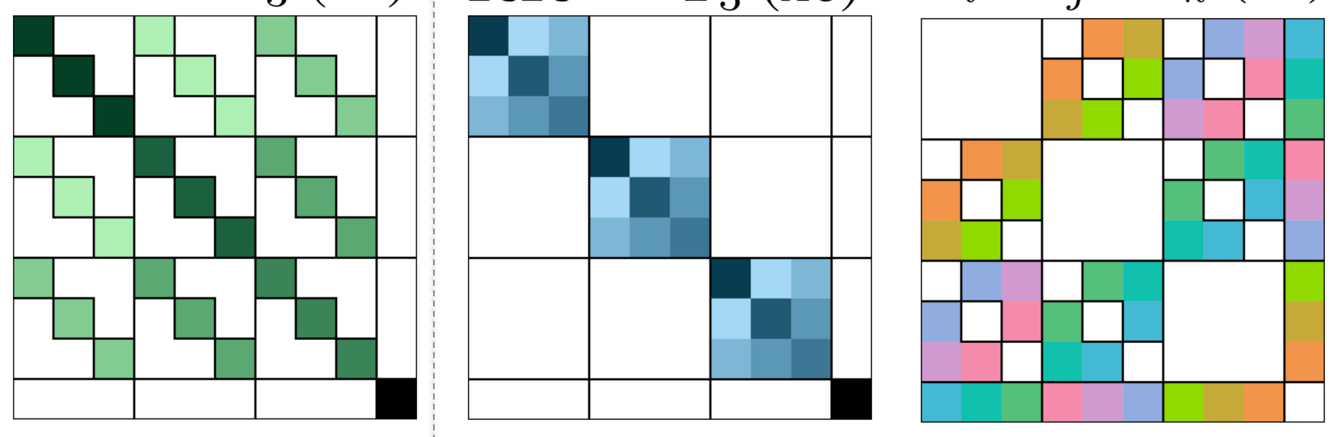
Usual constr.

$R^\top R = I_3$
$\det(R) = +1$

Not quadratic!

All quadratic constr.: Each color a constraint

Ort. Columns	Ort. Rows	Right hand
$R^\top R = I_3$ (x6)	$RR^\top = I_3$ (x6)	$c_i \times c_j = c_k$ (x9)



$\sum \lambda_i \tilde{P}_i$

Constr. in [2]

Our constraints (x21)

Empirical observation: If all (x21) constraints were added in ($\mathcal{D}$), there was *always* strong duality ($d^\star = f^\star$).

Experiments: % optimal solutions (*ours* 100%), *faster* than BnB

We evaluate how often a method attains the globally optimal solution:

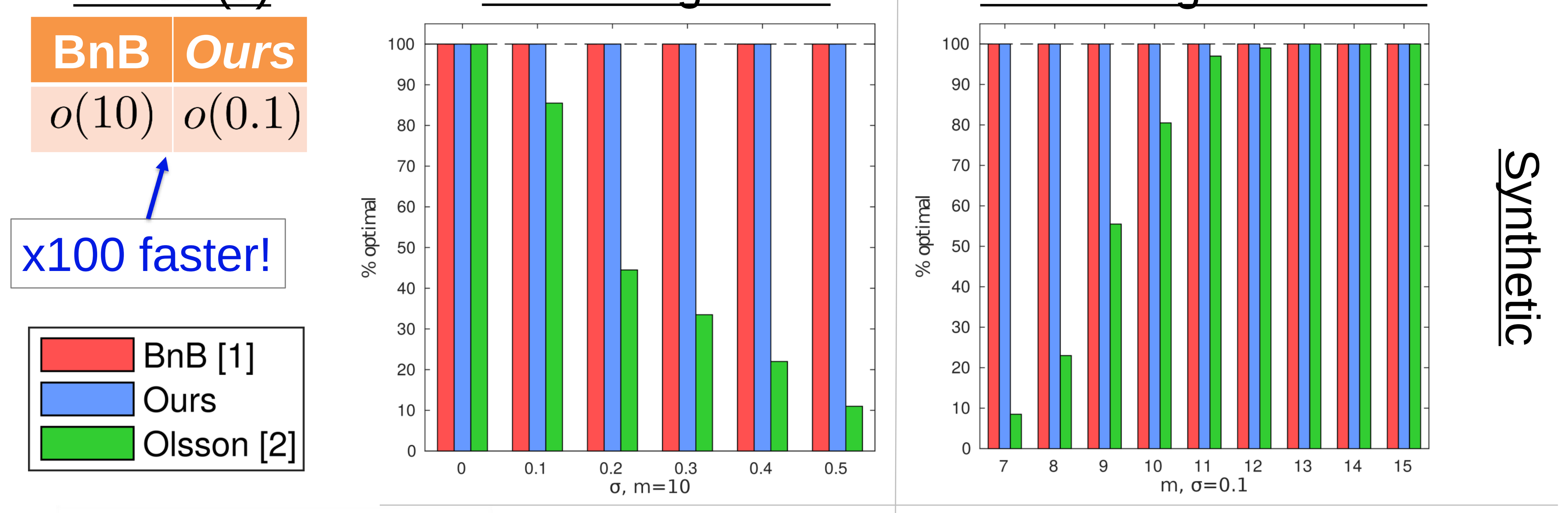
Times (s)

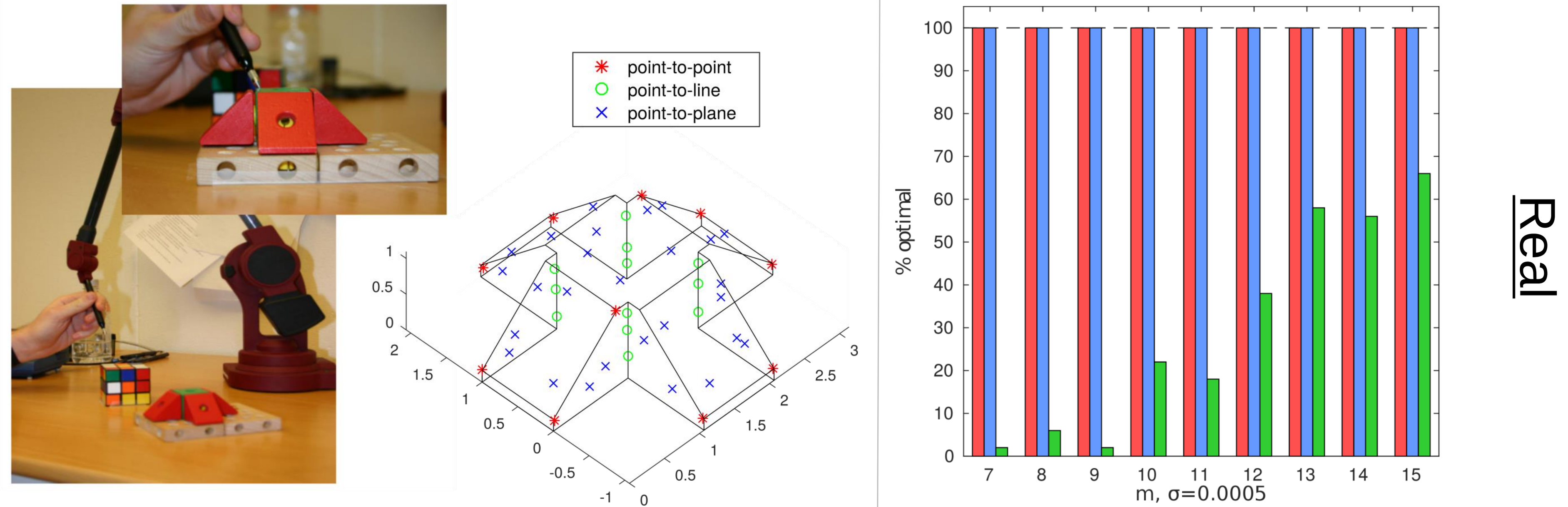
BnB	*Ours*
$o(10)$	$o(0.1)$

x100 faster!

Increasing noise

Increasing num. data





Conclusion

Empirically, we show the non-convexity of the constraint $R \in SO(3)$ can be circumvented when solving the studied registration problem.

Ongoing work: Theoretical guarantees | Faster SDP solver | Optimality verification | Multiple global minima | Robust registration

References:

[1] C. Olsson, F. Kahl and M. Oskarsson. *Branch-and-Bound Methods for Euclidean Registration Problems*. In IEEE Trans. Pattern Anal. Mach. Intell. (TPAMI), 2009.

[2] C. Olsson and A. Eriksson. *Solving Quadratically Constrained Geometrical Problems using Lagrangian Duality*. In Intl. Conf. Pattern Recognition (ICPR), 2008.