Generating 5D Light Fields in Scattering Media for Representing 3D Images

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Abstract

In this paper, we propose a novel method for displaying 3D images based on a 5D light field representation. In our method, the light fields emitted by a light field projector are projected into 3D scattering media such as fog. The intensity of light lays projected into the scattering media decreases because of the scattering effect of the media. As a result, 5D light fields are generated in the scattering media. The proposed method models the relationship between the 5D light fields and observed images, and uses the relationship for projecting light fields so that the observed image changes according to the viewpoint of observers. In order to achieve accurate and efficient 3D image representation, we describe the relationship not by using a parametric model, but by using an observation based model obtained from a point spread function (PSF) of scattering media. The experimental results show the efficiency of the proposed method.

1. Introduction

In recent years, 3D image representation is widely used in various fields, such as entertainment, medical systems and human computer interaction. In general, a 3D images are displayed by presenting different 2D images to different viewpoints. It can be achieved by using many techniques, such as parallax barriers [1, 5] and lenticular lenses [10]. However, these methods unfortunately cause inconsistency in depth perception of human visual systems. The human visual systems receive 3D informations not just from the disparity but also from the depth focus. Since the depth informations obtained from the disparity and the depth focus are not consistent to each other in these existing methods, the human observers feel sick when they observe 3D images represented by these methods.

The problem can be overcome, if the depth informations come from all the sensing systems are consistent to each other. For this objective, some methods have been proposed for displaying 3D objects directly in the 3D space [18, 15, 17, 11]. Rakkolainen et al. [17] used fog as a screen in the 3D space, and projected images directly on the fog screen. Although their method can project images directly in the 3D space, it only considers reflection on the surface of the scattering media. In other words, the fog in the 3D space was used not as a 3D screen but as a translucent 2D screen in their method. Although the fog screen has been extended for visualizing 3D objects [8, 24], all the existing methods simply used fog as 2D screens which scatter lights.

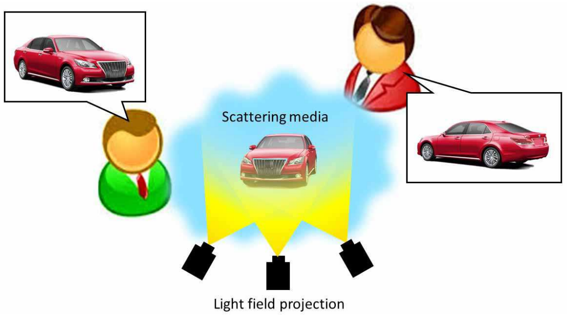


Figure 1. Displaying 3D images by light field projection toward a scattering medium

In this paper, we propose a new method for representing 3D objects using 3D scattering media, such as fog. In our method, a scattering medium is used not as a 2D screen which reflects lights on the surface of the scattering media, but as a 3D screen which scatters lights at the inside of the medium. As a result, 3D objects can be displayed directly in the 3D space as shown in Fig. 1.

For this objective, we use light field projection[6, 13, 2], which projects 4D light fields into the space. When we project a 4D light field into the scattering media, the lights in the light field are scattered and absorbed by the media, and their intensities change according to their depth in the scattering media. As a result, a 5D light field is generated in the scattering media. In this paper, we analyze the relationship between the 5D light fields and their observations in images. Based on this analysis, we project light fields from a light field projector, so that the observed images change according to the viewpoint of the observers. For this objective, we describe the relationship between the 5D light fields and observed images not by using a parametric model, but by using an observation based model based on a point spread function (PSF) of the scattering media.

By using the proposed method, different images can be presented to observers according to their viewpoints, and thus 3D objects can be displayed directly into the 3D space as shown in Fig.1.

2. Related Work

In recent years, light field displays and projectors have been studied extensively for representing 2D and 3D information [9, 23, 2, 6, 13]. Levoy and Hanrahan [9] proposed a method for rendering view dependent images by generating 4D light fields in the 3D space. More recently, Wetzstein et al. [22, 23] proposed a method for representing 4D light fields by using a layered display device which they called a tensor display. They showed that colored light fields can be generated by the subtractive color model of multiple LCD panels. The use of light field displays for wearable devices has also been studied recently. Lanman and Luebke [7] proposed light field displays for eyeglass devices, and Huang et al. [3] proposed a light field stereoscope. The applications of light field displays and projectors have also been studied. Sato et al. [19] proposed a method for modeling and synthesizing the appearance of objects by controlling multiple point light sources. Huang et al. [4] proposed a method for correcting visual aberrations by using 4D light field display.

Although these methods and systems can present light fields for various objectives, they only consider light fields in the 4D space. This is because all these methods assume that the intensity of light does not change along the light travel. This assumption can be applied to most of the cases, but it does not hold when the light is absorbed and scattered during the light travel. In this paper, we consider the light fields in scattering media, such as fog. In such scattering media, the lights are scattered and absorbed. As a result, the intensity of light changes along the light travel. In this case, the existing 4D light field based methods are no longer applicable. In this paper, we consider 5D light fields generated in the scattering media, and control them so that we can present arbitrary visual information toward observers at different viewpoints.

For representing visual information in scattering medias, fog screen has been invented by Rakkolainen et al. [16]. They showed that 2D fog screens can be used as efficient interactive display devices. The fog screen has also been used for visualizing 3D objects. Lee et al. [8] showed that 3D effects can be obtained by combining images projected onto two different planar fog screens. Yagi et al. [24] proposed a method for displaying different visual information at different viewpoints by using a cylindrical fog screen. Although their method can visualize different images to different viewpoints, they simply used strong directivity of light in scattering media and displayed different 2D images to different observers. That is, they assumed that the highest intensity of scattered light is visible just in front of the projector, and displayed different images to observers in front of different projectors through the scattering media. As a result, the viewpoints of the observer are fixed to the other side of the projectors in their method. Unlike their method, we in this paper consider 5D light fields generated by light field projectors and show that it is possible to visualize different visual information to arbitrary viewpoints.

The systematic analysis of light transport in scattering media has been conducted in recent years. Mukaigawa et al. [12] proposed a method for separating single scattering components from multiple scattering components in observed images. Tanaka et al. [21] proposed a method for recovering the inner structure of translucent objects by obtaining the direct component of scattering lights. Although the separation of lights is useful for analyzing objects, we need whole properties of the scattering media for visualizing 3D objects in the media. Thus, in this research we model the whole properties of scattering media by obtaining their PSF under light field projection.

3. Light Field in Scattering Media

If there is no obstacle in the 3D space, a light ray goes straight with a constant intensity, and the intensity L of each light ray is described by a 4D function as follows:

$$L = L^4(x, y, u, v) \tag{1}$$

where, x and y represent the horizontal and vertical position on a basis plane in the 3D space, through which a light ray passes, and u, and v represent the horizontal and vertical orientations of the light ray.

However, if the light ray goes into a scattering medium, such as fog and smoke, the light ray is reflected and absorbed at micro particles which exist in the scattering medium. As a result, the light ray is attenuated, and the intensity of light ray changes according to the position in the scattering medium. Thus, for representing a light field in the scattering medium, we need to consider a 5D function as follows:

$$L = L^5(x, y, z, u, v) \tag{2}$$

where, x, y and z represent a 3D position in the space.

For describing the attenuation of light in the scattering media, a parametric model based on Beer-Lambert law is often used [14, 20, 12]. The intensity of input light L projected into the scattering media decreases according to its travel distance in the scattering media, and the intensity of light L' after the travel with distance d is described as follows:

$$L' = L\sigma e^{-\tau d} \tag{3}$$

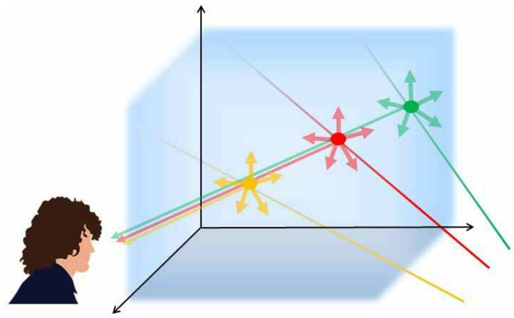


Figure 2. Observed intensity of 3D scattering is the sum of light rays directed to the viewpoint of the observer

where, σ is a scattering coefficient, and τ is an attenuation coefficient. If the orientation of the light ray and the orientation of the observer is different, the change in intensity becomes more complex, and the output light intensity L' is described as follows:

$$L' = L\sigma p(\theta)e^{-\tau d} \tag{4}$$

where, $p(\theta)$ is a phase function which describes the distribution of output light with respect to the relative orientation θ between the input light ray and the output light ray, and is modeled as follows:

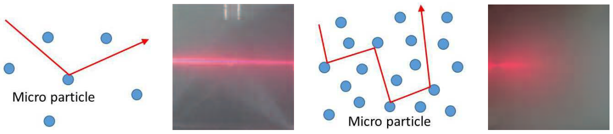
$$p(\theta) = \frac{1-g^2}{4\pi(1+g^2-2g\cos\theta)^{\frac{3}{2}}} \tag{5}$$

where, g is a scalar which represents the anisotropy of scattering, and takes a large value when the light scattering is strongly anisotropic, and takes zero when the light scattering is isotropic.

Although the parametric attenuation models shown in Eq.(4) and Eq.(5) are very useful and have been used in many existing works, they require accurate estimation of scattering parameters for describing precise light ray intensity in scattering media. Moreover, if the scattering parameters are not uniform in a scattering medium, these models are no longer applicable, since the estimation of nonuniform scattering parameters is very difficult in general.

If we consider the observation of the light field in the scattering media, we have additional difficulties. That is, an observed light ray comes not just from a single point but also from many points in the viewing direction in the scattering media as shown in Fig. 2.

Furthermore, if we have many particles in the scattering media, the observation becomes more complicated. If there are small number of particles in the media, the light rays are reflected just once by the particles as shown in Fig.3 (a) and (b). This is called a single scattering. However, if there are many particles in the media, the light rays are reflected many times as shown in Fig.3 (c) and (d). This is called a multiple scattering. When the multiple scattering is not negligible, the observed light in a viewing direction consists of not only single scattering lights but also multiple scattering lights as shown in Fig. 4.



(a) single scattering (b) example of (a) (c) multiple scattering (d) example of (c)

Figure 3. Single scattering and multiple scattering in scattering media

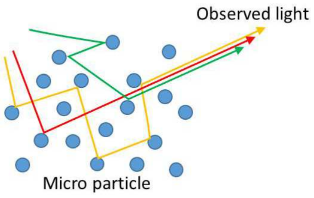


Figure 4. Observed light under multiple scattering

Since the observation of light fields is very complicated in scattering media as described above, parametric models such as Eq.(4) and Eq.(5) are not sufficient, and we need a more simple and powerful way to represent the observed light in the scattering media.

For representing the scattering characteristics in the scattering media more efficiently, and for describing the observed intensity more accurately, we in this paper describe the relationship between the projected light fields and the observed intensities by using not a parametric model but an observation based model as we describe in the next section.

4. PSF of Light Ray Scattering

Let us consider the case where a unit light ray $L(x,y,u,v) = 1$ is projected into a scattering medium from (x,y) toward (u,v) by using the light field projector. The light ray entered into the medium is attenuated and scattered, and then, light rays reflected toward an ith viewpoint O_i are observed as an image $\mathbf{P}^i(x,y,u,v)$ as shown in Fig.5. This image represents the amount of lights observed at the viewpoint O_i, when we project a unit light ray $L(x,y,u,v) = 1$ from (x,y) toward (u,v). Thus, it represents the scattering characteristics of the medium directly without using any parameters of the scattering medium. In this paper, we consider the observed image $\mathbf{P}^i(x,y,u,v)$ as the PSF (Point Spread Function) of a light ray $L(x,y,u,v)$ at the ith viewpoint. By changing the light rays $L(x,y,u,v)$ and observing their PSF $\mathbf{P}^i(x,y,u,v)$, we can obtain a set of PSFs of all the light rays, i.e. PSF of a light field, which directly represents the relationship between a projected light field and its observation in images.

If we fix the light field projectors and observers in the 3D

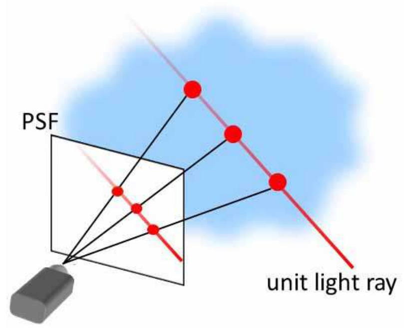


Figure 5. Point Spread Function (PSF) of a light ray in the scattering medium at a viewpoint

space, the position (x, y) and the orientation (u, v) do not need to be the real position and orientation of a light ray, but they can be the indices of the ray. For example, (x, y) can be the horizontal and vertical indices of a projector, and (u, v) can be the horizontal and vertical indices of a pixel in the projector image. In this case, $L(x, y, u, v)$ is a light ray projected from the (u, v)-th pixel in the (x, y)-th projector, and $\mathbf{P}^i(x, y, u, v)$ is its PSF in the ith camera. By considering the light field and its PSF in this way, we no longer need to calibrate light field projectors, i.e. light rays in the light field. This simplifies the setup of light field projectors greatly.

The scattering model based on the PSF can describe both the single scattering and the multiple scattering in the same manner. Furthermore, it can also describe light scattering in inhomogeneous media, which cannot be described by neither the single scattering model nor the multiple scattering model. In the inhomogeneous media, the scattering characteristics are not isotropic, but are different according to the position in the media. As a result, the existing parametric models of light scattering are no longer applicable. On the contrary, the PSF based model described in this section can represent complex light scattering in such inhomogeneous scattering media, and thus it is much more powerful than the existing parametric models.

Note, the PSF changes according to the viewpoints, and thus, we need to measure the PSF at every viewpoint in order to present arbitrary images to arbitrary viewpoints for displaying 3D images. Once the PSFs are obtained at all the viewpoints, we can use them for computing a light field for representing arbitrary images to arbitrary viewpoints as we explain in the next section.

5. Image Presentation in Scattering Media

5.1. Image Presentation for Single Viewpoint

We next consider a method for estimating projection images toward scattering media to represent arbitrary images to each viewpoint. At first, we consider the observation from a single viewpoint using the PSF model.

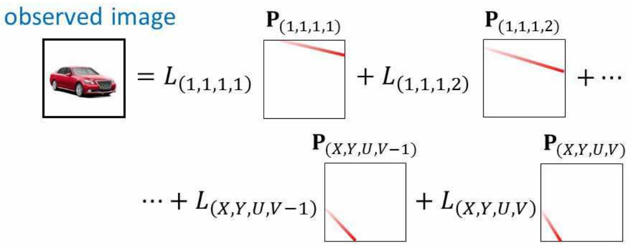


Figure 6. Representation of an observed image by using a set of PSFs

As described in section 3, we observe the sum of light rays scattered toward the viewpoint. Thus, the observed image can be represented by the sum of PSFs as shown in Fig.6. Let $\mathbf{P}^i(x, y, u, v)$ be the PSF of a light ray $L(x, y, u, v)$ at the ith viewpoint O_i. Then, the observed image $\mathbf{I}^i$ at O_i can be described as follows:

$$\begin{aligned}\mathbf{I}^i &= \sum_{x=1}^{X}\sum_{y=1}^{Y}\sum_{u=1}^{U}\sum_{v=1}^{V} L(x,y,u,v)\mathbf{P}^i(x,y,u,v) \\ &= \sum_{x,y,u,v}^{X,Y,U,V} L(x,y,u,v)\mathbf{P}^i(x,y,u,v) \qquad (6)\end{aligned}$$

where, X and Y are the number of horizontal and vertical position indices, and U and V are the number of horizontal and vertical orientation indices respectively.

Suppose we have an objective image $\mathbf{I'}^i$ for the ith viewpoint O_i. Then, a light field $\mathbf{L} = [L(1,1,1,1), \cdots, L(X,Y,U,V)]^\top$ for representing the objective image $\mathbf{I'}^i$ toward the ith viewpoint O_i can be estimated by minimizing the following cost function E^i:

$$E^i = \|\mathbf{I'}^i - \sum_{x,y,u,v}^{X,Y,U,V} L(x,y,u,v)\mathbf{P}^i(x,y,u,v)\|^2 \qquad (7)$$

Note, the intensity of light rays is limited in the real projectors, i.e. it should be positive and less than the maximum intensity of the projector. Therefore, the light field $\mathbf{L}$ can be estimated by solving the following conditional minimization problem:

$$\begin{gathered}\hat{\mathbf{L}} = \underset{\mathbf{L}}{\operatorname{argmin}}\, E^i \\ \text{subject to } \ 0 \leq L(x,y,u,v) \leq I_{\max}\end{gathered} \qquad (8)$$

where $I_{\max}$ is the maximum intensity of the projector.

By projecting the estimated light field $\hat{\mathbf{L}}$ from the light field projector into the scattering media, the objective image $\mathbf{I'}^i$ can be observed from the viewpoint O_i.

5.2. Image Presentation for Multiple Viewpoints

We next extend Eq.(8) for representing different images to multiple different viewpoints.

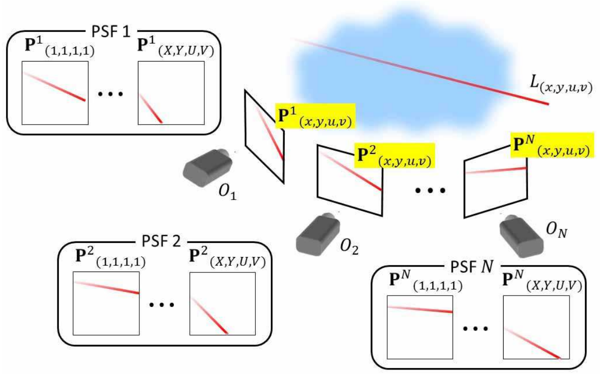


Figure 7. A set of PSFs for multiple viewpoints

Let us consider the case where N objective images $\mathbf{I}'^i$ $(i = 1, \cdots, N)$ are presented to N viewpoints O_i $(i = 1, \cdots, N)$. Suppose a set of PSFs for these N viewpoints is described by $\mathbf{P}^i(x, y, u, v)$ $(i = 1, \cdots, N)$ as shown in Fig.7. Then, the cost function E^i for each viewpoint can be described by Eq.(7). Thus, a light field $\mathbf{L}$ for presenting N different images to N different viewpoints can be estimated by solving the following conditional minimization problem:

$$\begin{array}{c} \hat{\mathbf{L}} = \underset{\mathbf{L}}{\operatorname{argmin}} \sum_{i=1}^{N} E^i \\ \text{subject to } \ 0 \leq L(x, y, u, v) \leq I_{\max} \end{array} \tag{9}$$

By projecting the light field $\hat{\mathbf{L}}$ from the light field projector, N objective images can be observed at N different viewpoints. Thus, we can achieve 3D image displaying by using the scattering media.

Note that the degrees of freedom (DoF) of the light field projected from the light field projector must be larger than the DoF of objective images for N viewpoints. That is, the number of projectors constituting the light field projector must be larger than the number of viewpoint, when the image size of a projector is the same as the image size of an objective image. If the DoF of the light field projector is larger than the DoF of objective images, Eq.(9) has many solutions. However, it is not a problem, since the observers can see objective images properly as long as the derived light field satisfies Eq.(9).

6. Prototype Light Field Projection System for Scattering Media

We next explain our prototype system built in this research for generating light fields in the scattering media and for displaying different images to different viewpoints.

As a scattering medium, we used water with white ink in a transparent cylindrical tank. The diameter of the cylindrical tank is 90 mm. For generating dense light fields in a scattering medium, we need to put many projectors around the scattering medium, so that their lights intersect in the scattering medium as shown in Fig. 8 (a). However, it is not easy to set and control many projectors simultaneously. Therefore, in this research, we put a concave lens array sheet in front of a projector. By using the concave lens array, the light rays projected from a single projector spread at each lens, and they intersect in the scattering medium as shown in Fig. 8 (b).

Since a single projector does not cover wide angle around the scattering medium, we put the lens array sheet around the cylindrical tank, and projected lights from two projectors as shown in Fig. 8 (c). This enables us to generate dense light fields in the scattering medium. The lens array sheet used in our system is D-45T of Techjam, which has 6 × 7 lenses in a single sheet. We used 2 × 4 of them for generating 8 bundles of light from 2 projectors. We consider them as 8 virtual projectors.

Fig. 9 (a) shows our prototype light field projection system built in this research, and Fig. 9 (b) shows the scattering medium with a concave lens array sheet. Although the light rays refracted by a curved lens are no longer those from a single light source, it does not matter in our method, since our method is based on the direct observation of PSF in scattering media, and it does not require a bundle of lights to be projected from a single point.

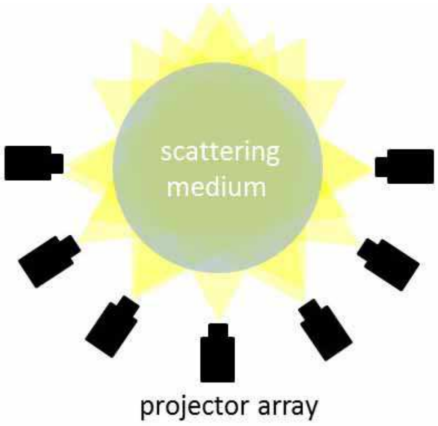
7. Experiment

We next show experimental results from the prototype light field projection system described in section 6. In this experiment, we presented three different images to three different viewpoints by using the proposed method. For observing these three images, we put three cameras around the prototype light field projection system.

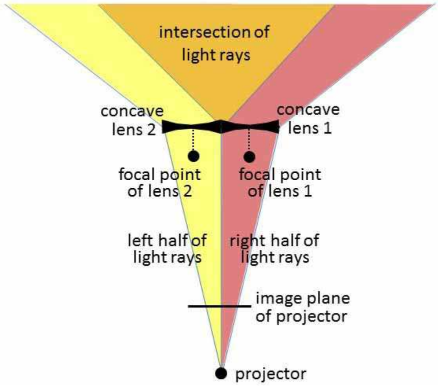
7.1. PSF Measurement

We first obtained the PSF of light rays in the scattering medium by using the prototype light field projection system. In this measurement, a unit light ray, i.e. a single pixel image, was projected into the scattering medium from each of 8 virtual projectors, and was observed by each camera as a PSF image at each viewpoint. Fig.10 (a) shows some example single pixel images projected from these projectors, and Fig.10 (b), (c) and (d) show PSFs observed in three cameras. In order to eliminate the effect of the ambient light, we took an ambient light image under black image projection, and subtracted it from PSF images.

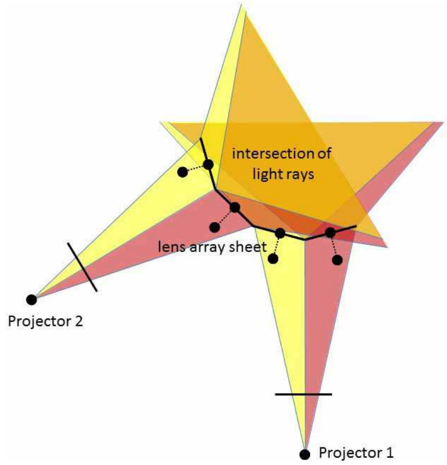
As shown in Fig. 10, PSFs in the scattering medium are elongated, and are very different from the standard PSF on the surface of opaque objects. Also, we find that the PSFs obtained at different viewpoints are different from each other. Since these PSFs are different from each other, we are able to present different images to different viewpoints.



(a) multiple projectors



(b) concave lens array



(c) multiple projectors with concave lens array

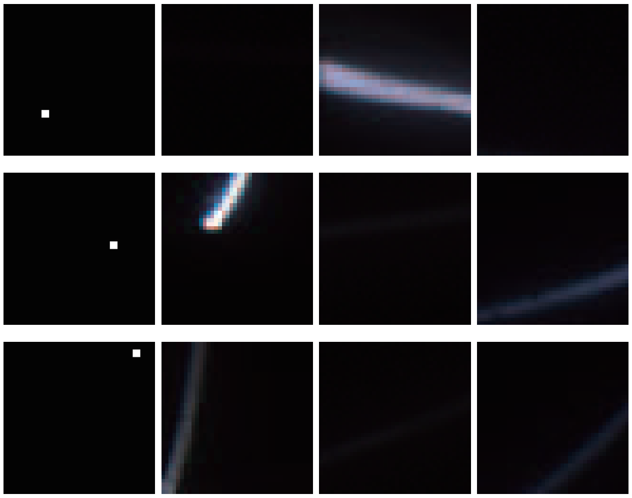
Figure 8. Light field projection for scattering media

7.2. Light Field Projection in Scattering Media

By using the PSFs obtained in section 7.1, we next presented three objective images shown in Fig. 11 to three different viewpoints. The light field projection images for presenting these three images were computed from the proposed method. The derived projection images for 8 virtual projectors are as shown in Fig.12. Surprisingly, the derived images for these 8 virtual projectors are very different from the original objective images as shown in Fig.12. This is because the PSFs in the scattering medium are very different from the ordinary PSF of opaque surface, and these PSFs are integrated intricately in the scattering medium as described in section 3.

(a) (b)

Figure 9. Prototype light field projection system and the scattering medium with a lens array sheet.



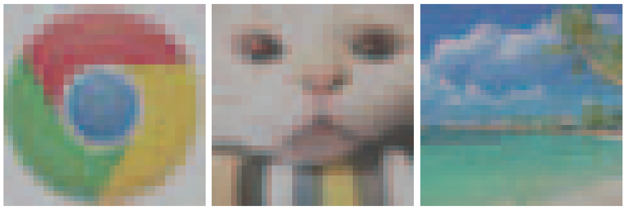
(a) projected image (b) PSF in camera 1 (c) PSF in camera 2 (d) PSF in camera 3

Figure 10. PSF measurement

By projecting these images from 8 virtual projectors, we projected a light field toward the scattering medium. The projected light field in the scattering medium was observed from three different viewpoints, i.e. three cameras. The observed images at these three viewpoints are as shown in Fig.13. As shown in Fig.13, the observed images of the proposed method are different according to the viewpoints, and are similar to the objective images shown in Fig.11. From these results, we find that the proposed method enables us to present different objective images to different viewpoints.

7.3. Evaluation

We next evaluate the proposed method by using synthetic images. In this evaluation, observed images at each view-



(a) viewpoint 1 (b) viewpoint 2 (b) viewpoint 3

Figure 11. Objective images of three viewpoints

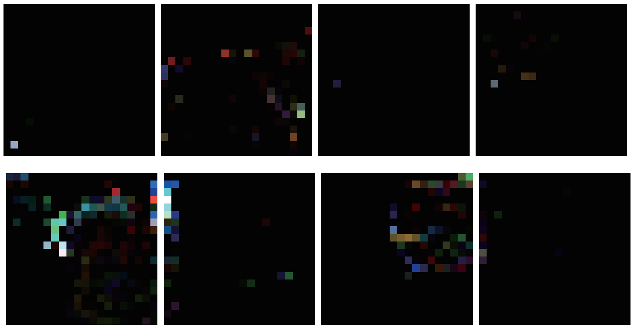


Figure 12. Images projected from 8 projectors.

(a) viewpoint 1 (b) viewpoint 2 (c) viewpoint 3

Figure 13. Observed results from the proposed method

point were synthesized by using the PSFs measured in section 7.1 by using the prototype light field projection system, and the error of observed images was computed comparing with the original objective images. The number of viewpoints was changed from one to three, and the relationship between the number of viewpoints and the accuracy of observed images was evaluated. Fig. 14 shows the result. As shown in Fig. 14, if we increase the number of viewpoints, the RMSE of observed images becomes large. This is because the DOF of light field becomes relatively small comparing with the DOF of projected images, and the influence of the intensity limitation of projector and the dependency of PSFs becomes relatively large, when we present more images. For avoiding this problem and presenting more images accurately, we need to use more projectors, and expand the DOF of light field.

Finally, we compare the proposed method with the existing projection method which projects objective images directly onto the surface of scattering media. In this comparison, we used PSFs measured from the real environment in section 7.1 for generating images observed at three viewpoints. In the proposed method, we computed projection images from the PSFs and projected them toward the scattering medium. In the existing method, we simply projected objective images toward the scattering medium. The images observed at three viewpoints are as shown in Fig.15 (a) and (b).

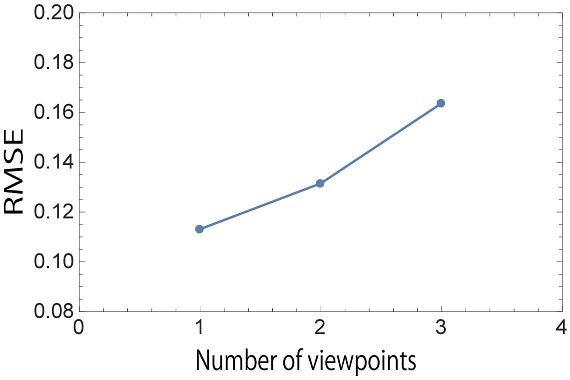


Figure 14. Relationship between the number of viewpoints and the accuracy of observed images. The range of image intensity is [0–1].

(a) proposed method

(b) existing method

Figure 15. Comparison of the proposed method with the existing direct projection.

As shown in Fig.15 (b), the simple projection of objective images toward the scattering medium does not provide us proper observation of objective images, while the proposed method can do it as shown in Fig.15 (a). Although the proposed method is still not perfect, the advantage of the proposed method is obvious in these images.

8. Conclusions

In this paper, we proposed a novel method for representing 3D images by controlling the 5D light fields generated in scattering media. The scattering media represent images not on their surface but at the inside of the media because of light ray scattering. Since the light rays are attenuated in the scattering media, 5D light fields are generated in the media.

For controlling the 5D light fields in the scattering media, we used a light field projector. The light rays emitted from the light field projector enter into the media and scattered. In this research, the light ray scattering was represented by a set of PSFs of the light rays observed at multiple viewpoints. By using the PSF model, we proposed a method for generating light fields to present arbitrary images toward arbitrary viewpoints.

The experimental results show that the proposed method can present different images to different viewpoint, and can represent 3D images in the scattering media. Unlike the parametric model based methods, the proposed method can represent 3D visual information in the scattering media even if the media are inhomogeneous.

The direct observation of the PSF of scattering media, which utilized in this paper, does not require the calibration of projector systems and does not need to estimate scattering parameters of the media. Thus the setup of the projection system is very easy, even if the target scattering media have complex scattering properties. However, the observation of PSF takes long time, and thus we need to investigate fast methods for obtaining the PSF of scattering media in our future work.

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