Supplemental Figures: Results for Various Color-image Completion

Anonymous authors

Comparison with various methods in color-image completion

In this supplemental document, we show the results of color-image completion by using various methods with explanations of the technical overview of the state-of-the-art methods. Table I summarizes the optimization concepts of the state-of-the-art tensor completion methods: high accuracy low-rank tensor completion (HaLRTC) [3], total variation regularization (TV reg.) [4], [6], low-rank and TV regularization (LRTV reg.) [4], simultaneous tensor decomposition and completion [1], smooth PARAFAC tensor completion with quadratic variation (SPCQV) [5], and the proposed method.

In HaLRTC, we minimize the tensor nuclear norm (tNN), which is defined as sum of matrix nuclear norm (NN) for all n-th matricizations of $\boldsymbol{\mathcal{X}}$, under the constraint of data consistency $\boldsymbol{\mathcal{P}}_\Omega(\boldsymbol{\mathcal{T}}) = \boldsymbol{\mathcal{P}}_\Omega(\boldsymbol{\mathcal{X}})$. In TV reg., we minimize the tensor total variation (tTV), which is an extension of matrix total variation (TV), under the constraint of data consistency. LRTV simultaneously minimizes tNN and tTV under the constraint of data consistency. Above three methods are formulated as convex optimization problems, and can be solved by primal-dual splitting (PDS) algorithm [2]. Since LRTV method proposed in [4] is a generalization of both HaLRTC and TV reg. methods and PDS algorithm is employed for optimization, we tried above three methods via PDS algorithm with appropriate hyper-parameter settings in LRTV formulation. In STDC, a Tucker decomposition problem with regularizations for a core tensor and factor matrices is considered. Factor matrices in STDC are imposed to small nuclear-norm to minimize the tensor rank, and factor prior imposes a kind of similarity of factor vectors each other. The optimization problem of STDC is non-convex, and it is solved by using augmented Lagrangian method. According to [1], the convergence of STDC algorithm is not theoretically guaranteed. In SPCQV, a PARAFAC decomposition problem with regularizations for factor vectors is considered. The PARAFAC decomposition consists of multiple rank-1 tensors, and each rank-1 tensor is constructed by N factor vectors. In SPCQV method, individual factor vectors are imposed to be smoothly variated, and number of rank-1 tensors is minimized. The optimization problem of SPCQV is also non-convex, and it is solved by using hierarchical alternating least squares (HALS) algorithm. Unlike the STDC, SPCQV has a monotonic non-increasing property of cost function and the convergence to stationary point is guaranteed. In the proposed method, a low-rank Tucker decomposition problem in embedded space is simply considered. In general, the optimization problem is non-convex, however, monotonic convergence property is guaranteed based on auxiliary function based optimization algorithm.

Figure 1 shows eight benchmark images used in this experiments. For each image, we generate eight types of incomplete images: (a) 50%, (b) 70%, (c) 90%, (d) 95%, and (e) 99% random voxel missing, (f) 11 continuous vertical slices missing, (g) cross shape occlusion with 50% random voxel missing, and (h) random vertical/horizontal slices missing. Thus, totally, 64 incomplete images were generated. For each incomplete image, we applied six tensor completion methods: HaLRTC, TV reg., LRTV reg., STDC, SPCQV, and the proposed method. Hyper parameters for all methods were tuned from several candidates, and employed the best settings. Figures 2-9 show the results of this experiments. First three convex methods recovered them for only low missing ratio cases. STDC outperformed the three convex methods, however, it failed to recover 95% and 99% missing cases and slice missing cases, and sometimes the algorithm did not converge. SPCQV successfully recovered random missing cases, however, it failed to recover slice missing cases. The proposed method obtained more clear results for random missing cases compared with SPCQV, and successfully recovered slice missing cases.

TABLE I: Optimization concepts

name	minimization	constraints
HaLRTC [3]	tensor nuclear norm (tNN): $\|\|\boldsymbol{\mathcal{X}}\|\|_{LR}$	$\boldsymbol{\mathcal{P}}_\Omega(\boldsymbol{\mathcal{T}}) = \boldsymbol{\mathcal{P}}_\Omega(\boldsymbol{\mathcal{X}})$
TV reg. [4], [6]	tensor total variation (tTV): $\|\|\boldsymbol{\mathcal{X}}\|\|_{TV}$	$\boldsymbol{\mathcal{P}}_\Omega(\boldsymbol{\mathcal{T}}) = \boldsymbol{\mathcal{P}}_\Omega(\boldsymbol{\mathcal{X}})$
LRTV reg. [4]	(tNN) + (tTV)	$\boldsymbol{\mathcal{P}}_\Omega(\boldsymbol{\mathcal{T}}) = \boldsymbol{\mathcal{P}}_\Omega(\boldsymbol{\mathcal{X}})$
STDC [1]	(NN of factor matrices) + (l2-norm of core tensor) + (factor prior)	Tucker decomposition + $\boldsymbol{\mathcal{P}}_\Omega(\boldsymbol{\mathcal{T}}) = \boldsymbol{\mathcal{P}}_\Omega(\boldsymbol{\mathcal{X}})$
SPCQV [5]	(number of rank-1 tensors) + (quadratic variation of factor vectors)	PARAFAC decomposition + $\boldsymbol{\mathcal{P}}_\Omega(\boldsymbol{\mathcal{T}}) = \boldsymbol{\mathcal{P}}_\Omega(\boldsymbol{\mathcal{X}})$
Proposed	(sum of multi-linear tensor ranks of $\boldsymbol{\mathcal{X}}_H$)	$\boldsymbol{\mathcal{P}}_\Omega(\boldsymbol{\mathcal{T}}) = \boldsymbol{\mathcal{P}}_\Omega(\mathcal{H}^{-1}(\boldsymbol{\mathcal{X}}_H))$

References

[1] Y.-L. Chen, C.-T. Hsu, and H.-Y. Liao. Simultaneous tensor decomposition and completion using factor priors. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 36(3):577–591, 2014.

[2] L. Condat. A primal–dual splitting method for convex optimization involving Lipschitzian, proximable and linear composite terms. *Journal of Optimization Theory and Applications*, 158(2):460–479, 2013.

Fig. 1: Eight benchmark images

[3] J. Liu, P. Musialski, P. Wonka, and J. Ye. Tensor completion for estimating missing values in visual data. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 35(1):208–220, 2013.
[4] T. Yokota and H. Hontani. Simultaneous visual data completion and denoising based on tensor rank and total variation minimization and its primal-dual splitting algorithm. In *Proceedings of CVPR*, pages 3732–3740, 2017.
[5] T. Yokota, Q. Zhao, and A. Cichocki. Smooth PARAFAC decomposition for tensor completion. *IEEE Transactions on Signal Processing*, 64(20):5423–5436, 2016.
[6] M. Zhu and T. Chan. An efficient primal-dual hybrid gradient algorithm for total variation image restoration. *UCLA CAM Report*, pages 08–34, 2008.

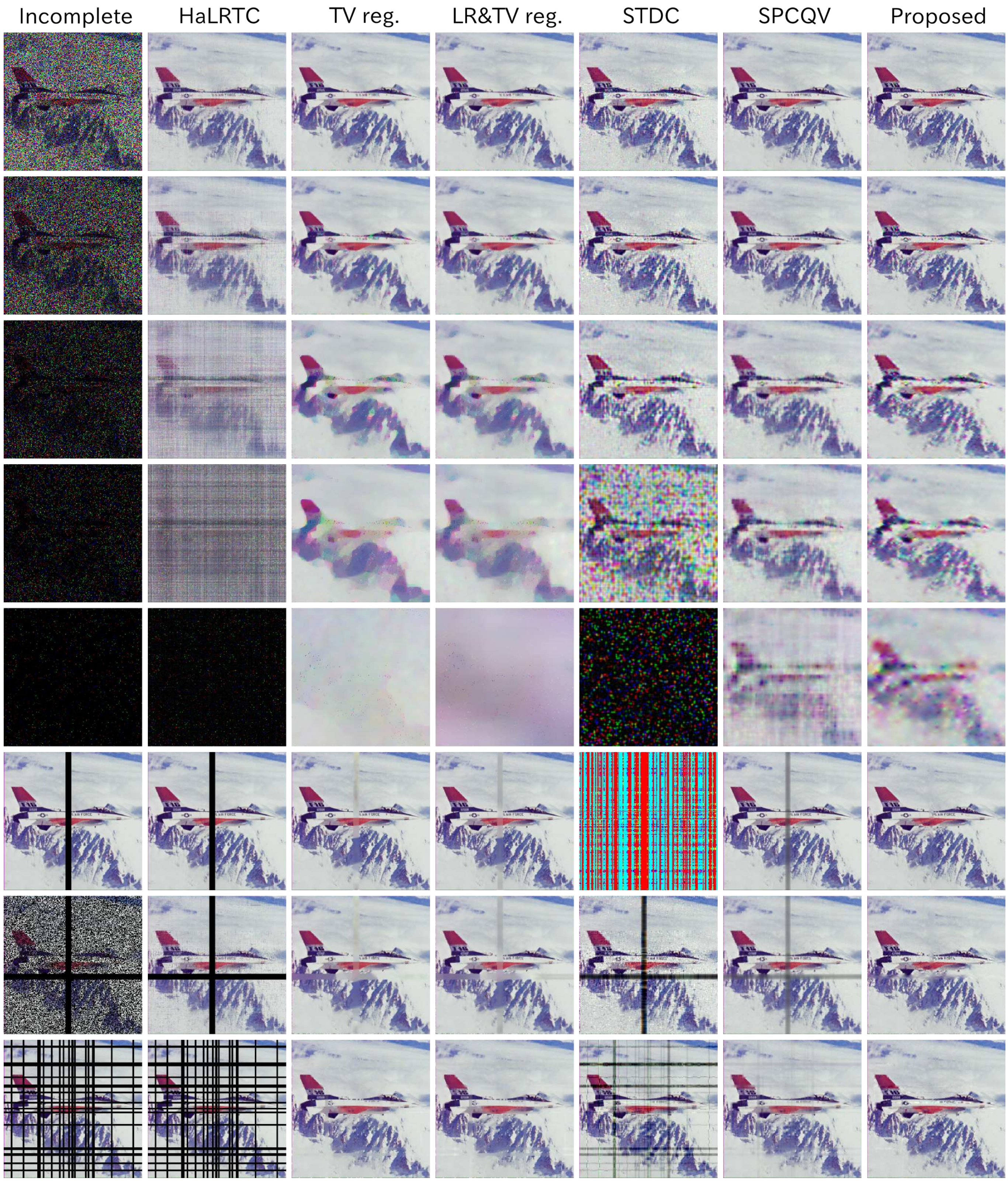

Fig. 2: Results in 'Airplain'

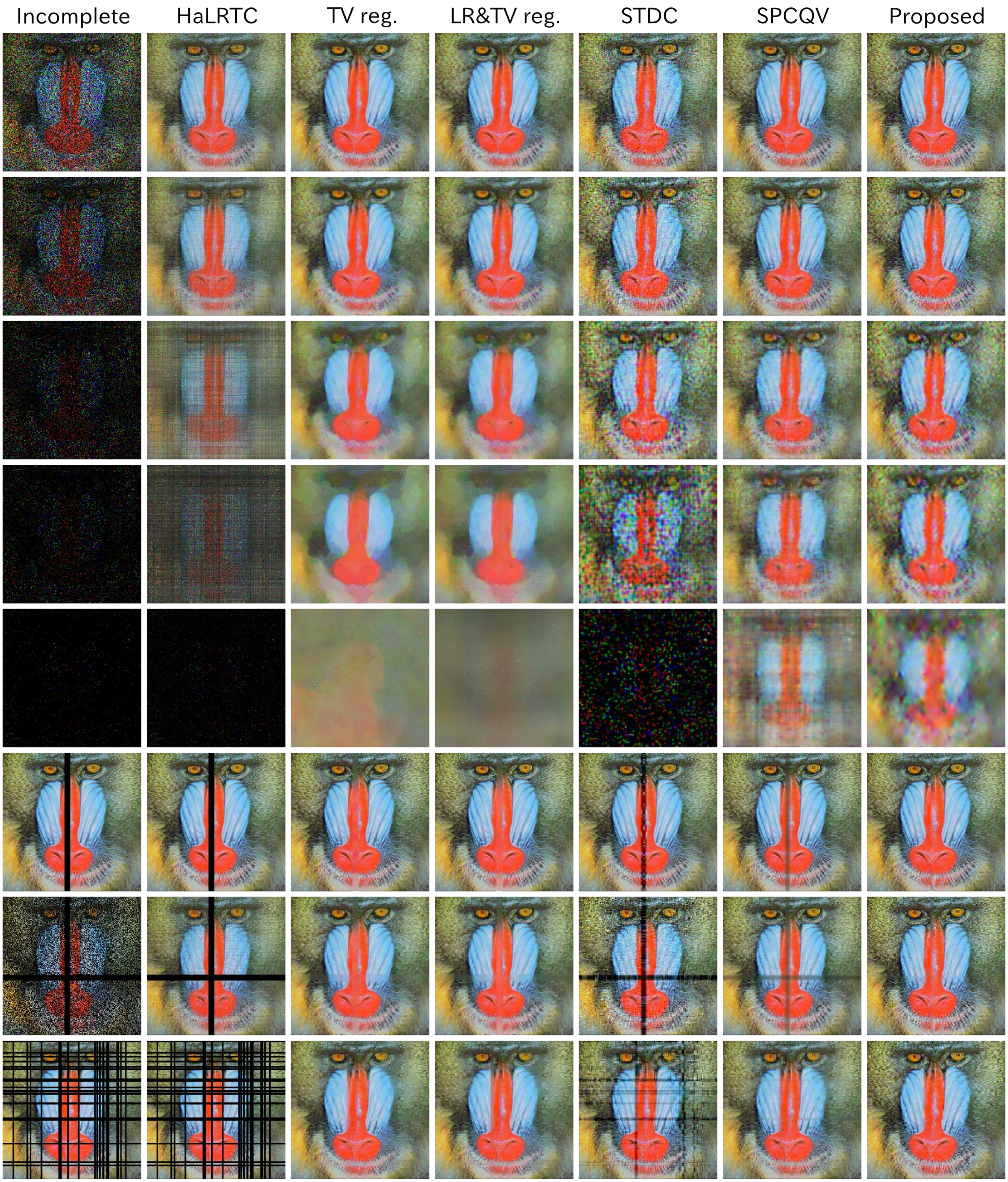

Fig. 3: Results in 'Baboon'

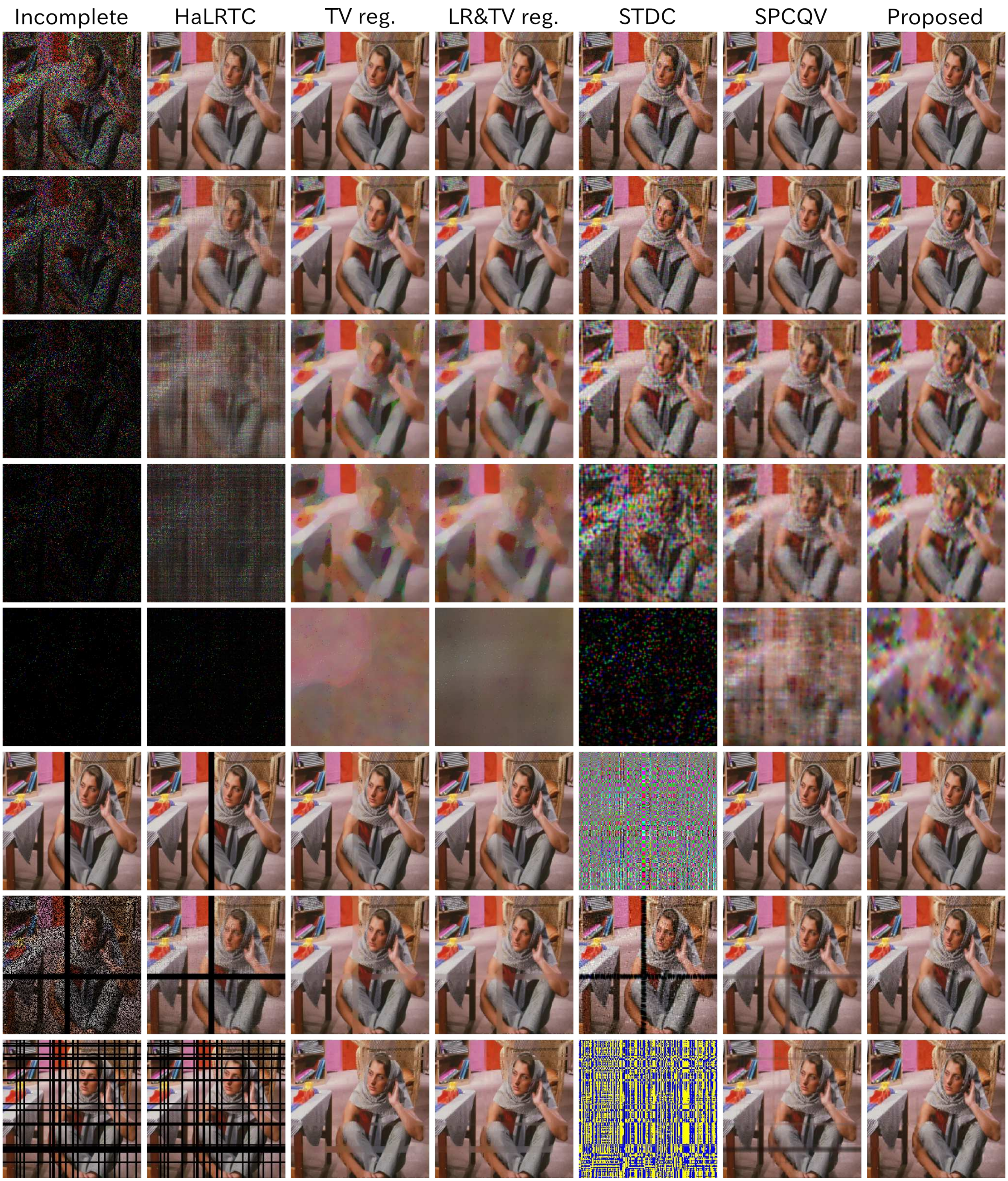

Fig. 4: Results in ‘Barbara’

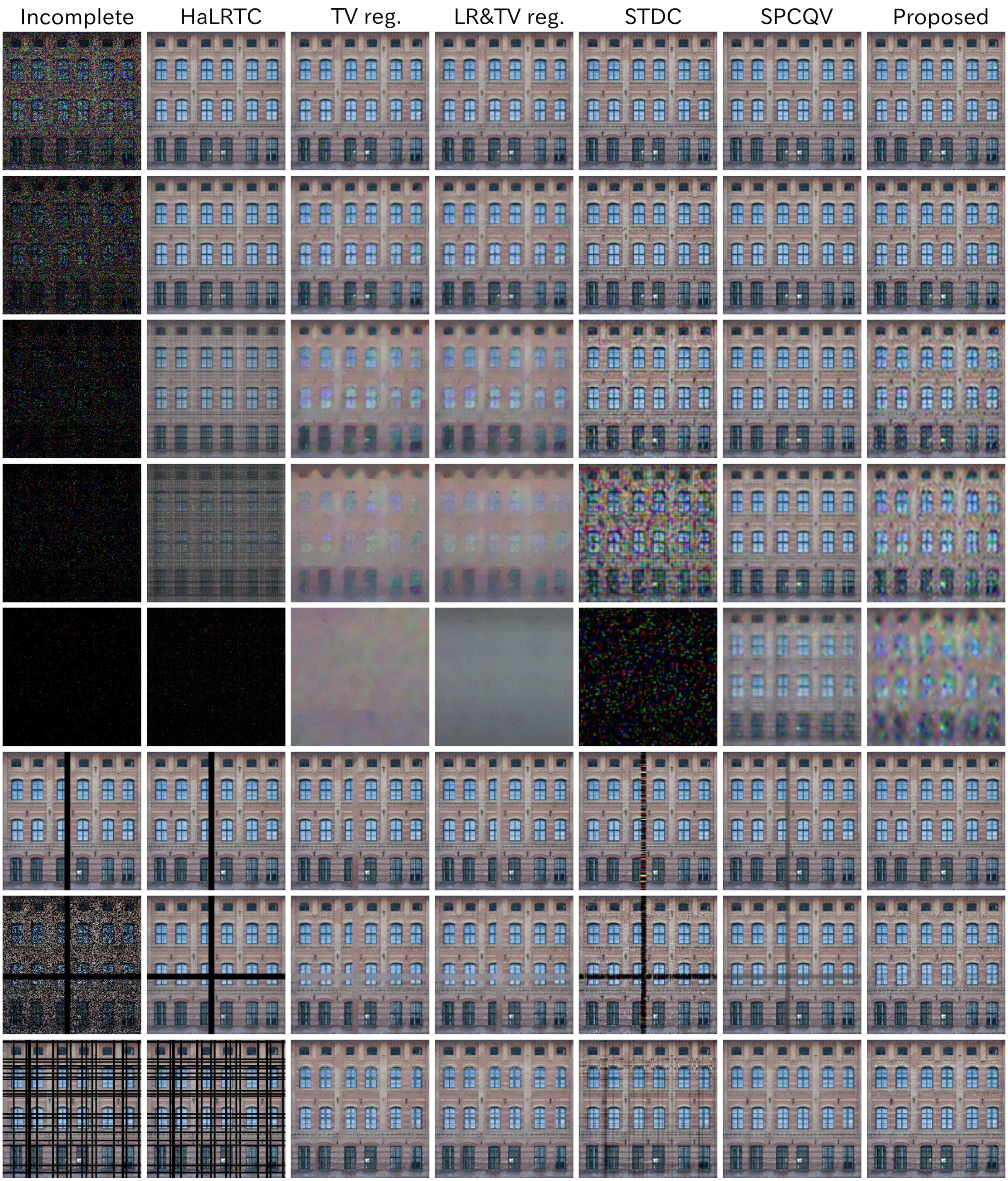

Fig. 5: Results in ‘Facade’

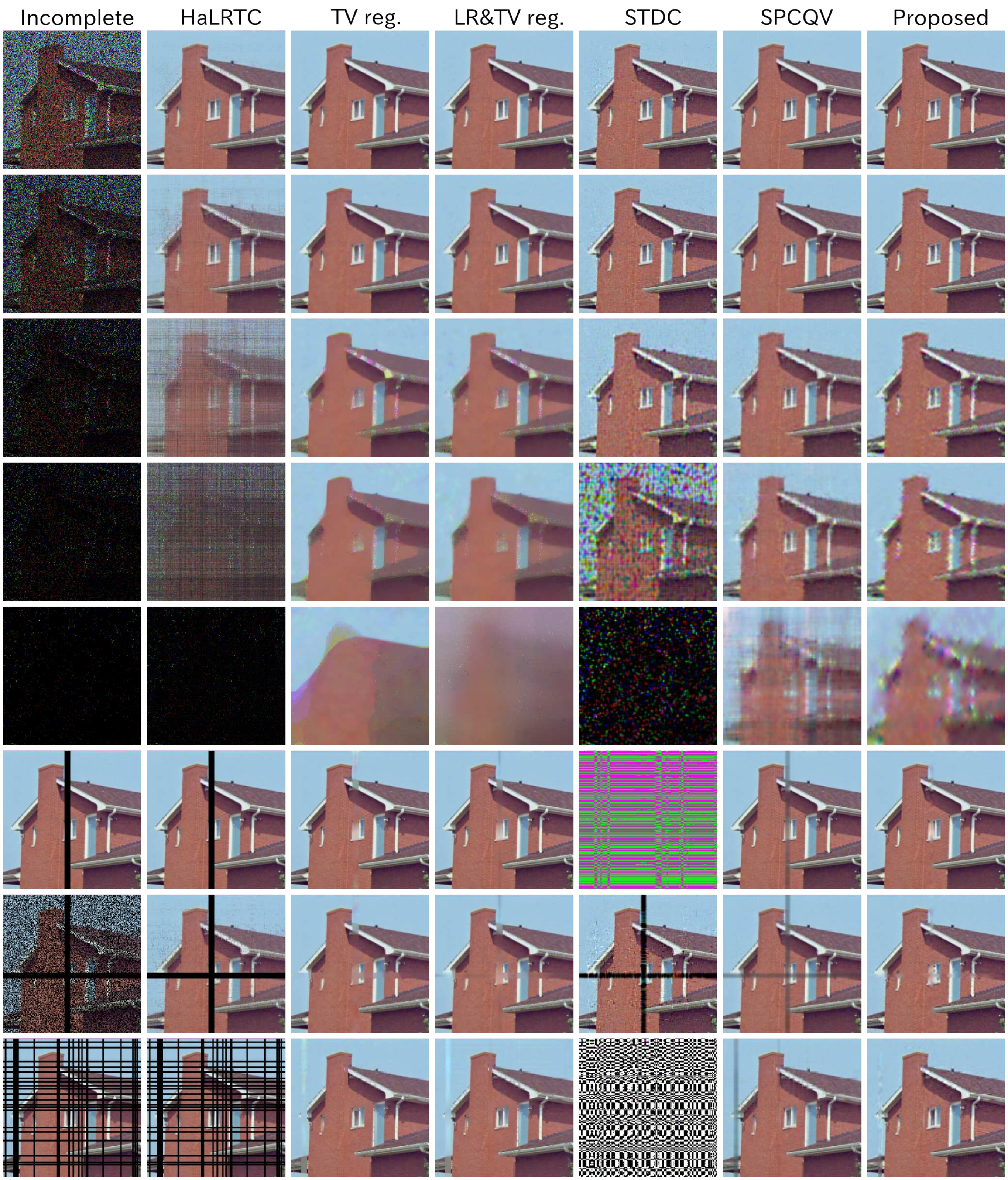

Fig. 6: Results in ‘House’

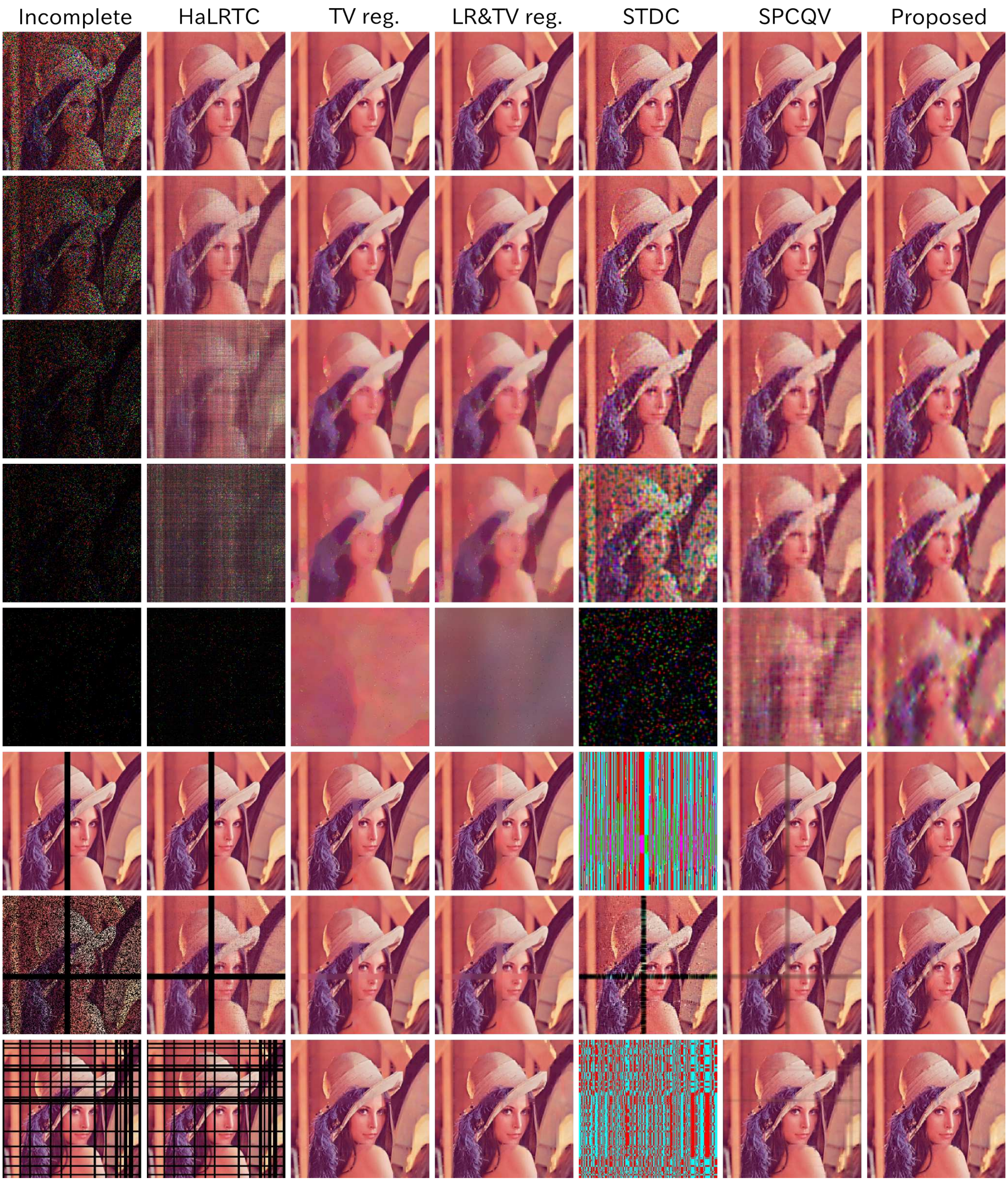

Fig. 7: Results in ‘Lena’

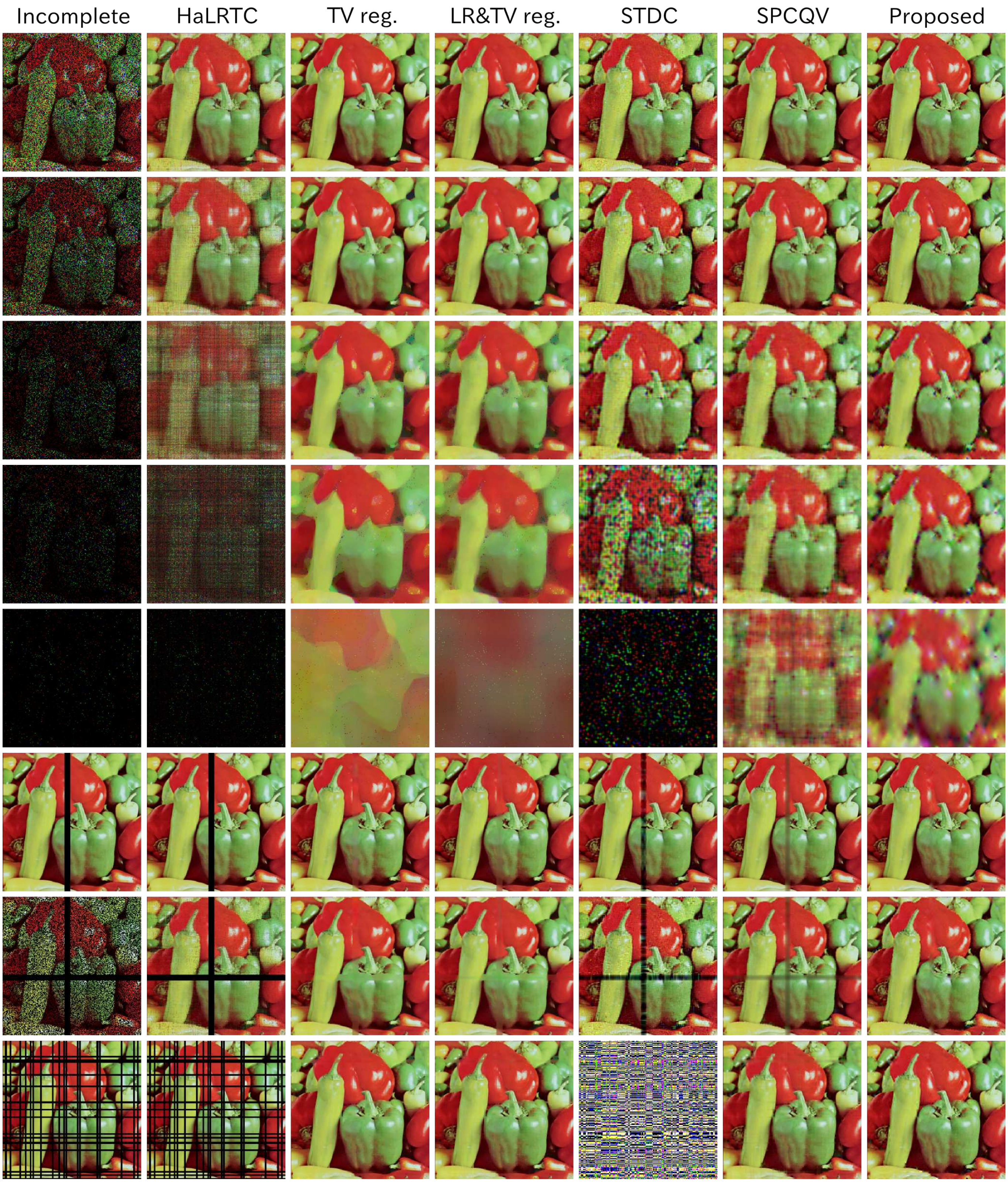

Fig. 8: Results in 'Peppers'

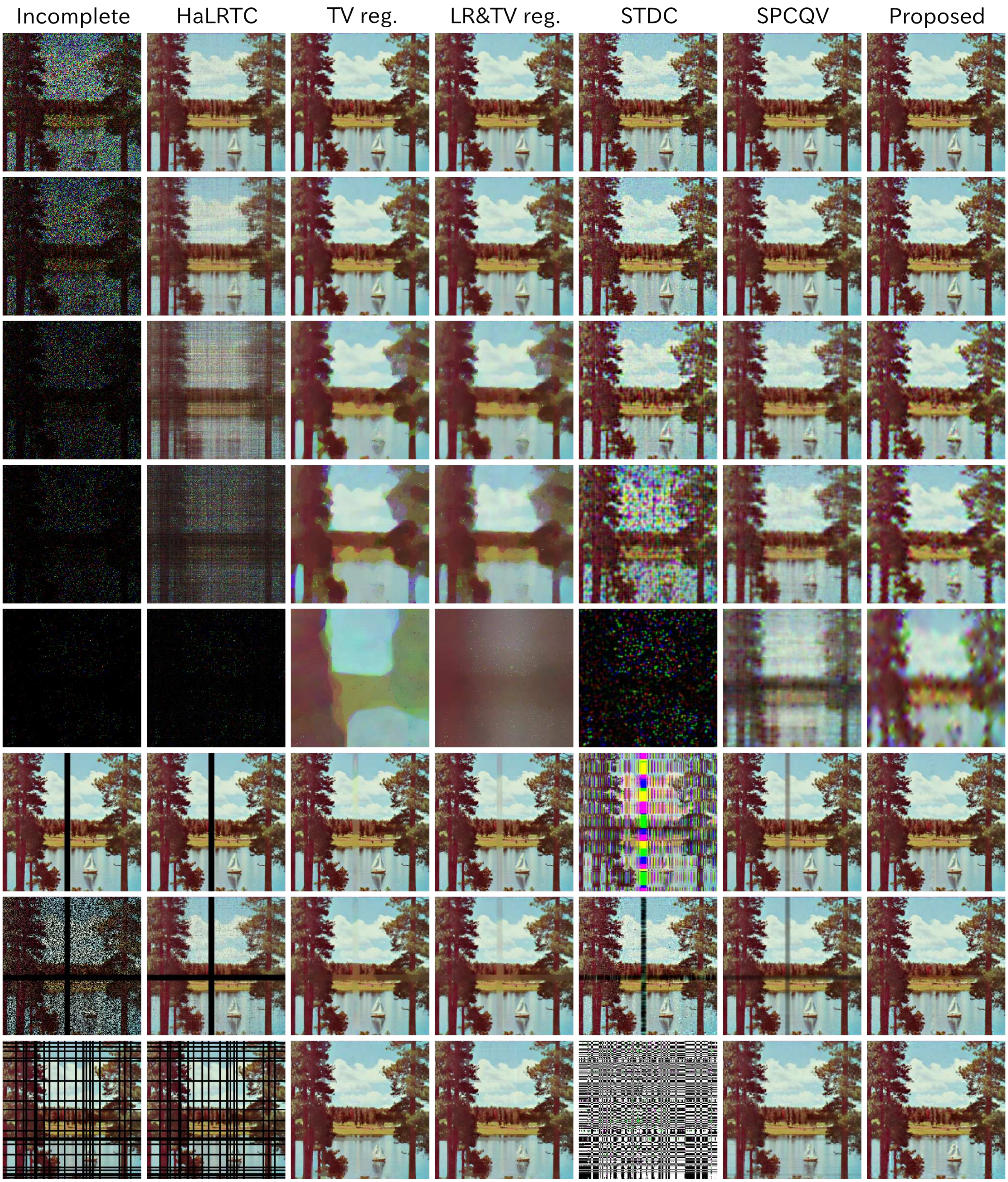

Fig. 9: Results in ‘Sailboat’