Supplementary Material

High-speed Tracking with Multi-kernel Correlation Filters

Ming Tang[1], Bin Yu[1], Fan Zhang[2], and Jinqiao Wang[1]
[1]National Lab of Pattern Recognition, Institute of Automation, CAS, Beijing 100190, China
[2]School of Info. & Comm. Eng., Beijing University of Posts and Telecommunications

Abstract

This supplementary material includes

1. *The experimental results on OTB2015.*
2. *The proof of Theorem 1.*

1. Experimental Results on OTB2015

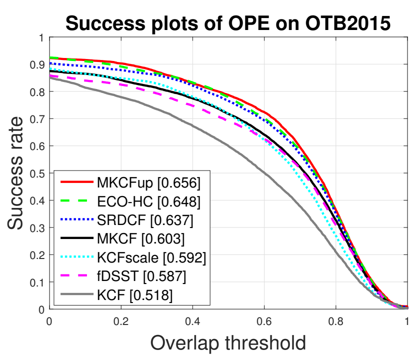

Figure 1. The success plot of MKCFup, KCF, KCFscale, MKCF, SRDCF, fDSST, and ECO_HC on small move sequences of OTB2015. The AUCs of the trackers on the sequences are reported in the legends.

2. Proof of Theorem 1

In the extension of MKCF with upper bound, to optimize the unconstrained problem

$$\min_{\boldsymbol{\alpha}_p,\mathbf{d}_p} F_p(\boldsymbol{\alpha}_p,\mathbf{d}_p), \tag{1}$$

we achieve that

$$\boldsymbol{\alpha}_p = \left(\sum_{j=1}^{p}\sum_{m=1}^{M}\beta_m^j\left((d_{m,p}\mathbf{K}_m^j)^2+\lambda d_{m,p}\mathbf{K}_m^j\right)\right)^{-1}\cdot \sum_{j=1}^{p}\sum_{m=1}^{M}\beta_m^j d_{m,p}\mathbf{K}_m^j\mathbf{y}_c, \tag{2}$$

and

$$d_{m,p} = \frac{d_{m,p}^N}{d_{m,p}^D}, \tag{3}$$

where

$$d_{m,p}^N = (1-\gamma_m)d_{m,p-1}^N + \gamma_m(\mathbf{K}_m^p\boldsymbol{\alpha}_p)^\top(2\mathbf{y}_c-\lambda\boldsymbol{\alpha}_p),$$
$$d_{m,p}^D = (1-\gamma_m)d_{m,p-1}^D + 2\gamma_m(\mathbf{K}_m^p\boldsymbol{\alpha}_p)^\top(\mathbf{K}_m^p\boldsymbol{\alpha}_p),$$

when $p>1$. If $p=1$, then

$$d_{m,1}^N = (\mathbf{K}_m^1\boldsymbol{\alpha}_1)^\top(2\mathbf{y}_c-\lambda\boldsymbol{\alpha}_1),$$
$$d_{m,1}^D = (\mathbf{K}_m^1\boldsymbol{\alpha}_1)^\top(\mathbf{K}_m^1\boldsymbol{\alpha}_1).$$

To simplify the denotation, in the proof, $d_{m,p}$ expresses the kernel weight $d_{m,p}^t$ of the t^{th} iteration of $\boldsymbol{\alpha}_p$ and $\mathbf{d}_p$.

2.1. Proof of First Conclusion

According to Eq. (2), we set $\boldsymbol{\alpha}_p = \mathbf{D}_p^{-1}\mathbf{N}_p\mathbf{y}_c$, where

$$\mathbf{D}_p = \sum_{j=1}^{p}\sum_{m=1}^{M}\beta_m^j\left((d_{m,p}\mathbf{K}_m^j)^2+\lambda d_{m,p}\mathbf{K}_m^j\right)$$

and

$$\mathbf{N}_p = \sum_{j=1}^{p}\sum_{m=1}^{M}\beta_m^j d_{m,p}\mathbf{K}_m^j.$$

It is clear that both $\mathbf{D}_p$ and $\mathbf{N}_p$ are positive definite because $\beta_m^j>0$, $d_{m,p}>0$, $\lambda>0$, and $\mathbf{K}_m^j$ is positive definite. Because $\mathbf{K}_m^j$ is circulant Gram matrix, we have $\mathbf{K}_m^j = \mathbf{U}\boldsymbol{\Sigma}_m^j\mathbf{U}^H$, where $\mathbf{U}=\frac{1}{\sqrt{l}}\mathbf{F}_l^{-1}$ and $\mathbf{F}_l$ is the 1-D discrete Fourier transform matrix [**?**]. Because the linear combination of circulant matrices is also circulant, we have

$$\mathbf{D}_p = \mathbf{U}\left(\sum_{j=1}^{p}\sum_{m=1}^{M}\beta_m^j\left(d_{m,p}^2(\boldsymbol{\Sigma}_m^j)^2+\lambda d_{m,p}\boldsymbol{\Sigma}_m^j\right)\right)\mathbf{U}^H$$

and

$$\mathbf{N}_p = \mathbf{U}\left(\sum_{j=1}^{p}\sum_{m=1}^{M}\beta_m^j d_{m,p}\boldsymbol{\Sigma}_m^j\right)\mathbf{U}^H.$$

Let $\mathbf{\Sigma}_m^j = \mathbf{diag}\left(\sigma_{m,1}^j, \dots, \sigma_{m,l}^j\right)$, $\sigma_{m,n}^j > 0$, $n = 1, \dots, l$. Then the n^{th} eigenvalue of $\mathbf{D}_p^{-1}\mathbf{N}_p$ is

$$\sigma_{\boldsymbol{\alpha}_p,n} \equiv \frac{\sum_{j=1}^p \sum_{m=1}^M \beta_m^j d_{m,p} \sigma_{m,n}^j}{\sum_{j=1}^p \sum_{m=1}^M \beta_m^j d_{m,p} \sigma_{m,n}^j (d_{m,p}\sigma_{m,n}^j + \lambda)} = (\lambda + b_n)^{-1},$$

where

$$b_n = \frac{\sum_{j=1}^p \sum_{m=1}^M \beta_m^j d_{m,p}^2 (\sigma_{m,n}^j)^2}{\sum_{j=1}^p \sum_{m=1}^M \beta_m^j d_{m,p} \sigma_{m,n}^j}.$$

It is clear that $b_n > 0$.

According to Eq. (3), we also have

$$\begin{aligned} d_{m,p}^N &= \sum_{j=1}^p \beta_m^j (\mathbf{K}_m^j \boldsymbol{\alpha}_p)^\top (2\mathbf{y}_c - \lambda \boldsymbol{\alpha}_p) \\ &= \mathbf{y}_c^\top \sum_{j=1}^p \beta_m^j \mathbf{N}_p \mathbf{D}_p^{-1} \mathbf{K}_m^j (2\mathbf{I} - \lambda \mathbf{D}_p^{-1}\mathbf{N}_p)\mathbf{y}_c \\ &= \mathbf{y}_c^\top \mathbf{D}_{m,p}^N \mathbf{y}_c, \end{aligned}$$

where $\mathbf{D}_{m,p}^N = \mathbf{N}_p \mathbf{D}_p^{-1} \sum_{j=1}^p \beta_m^j \mathbf{K}_m^j (2\mathbf{I} - \lambda \mathbf{D}_p^{-1}\mathbf{N}_p)$, and its n^{th} eigenvalue is

$$\sigma_{m,p,n}^N = \sigma_{\boldsymbol{\alpha}_p,n}(2 - \lambda\sigma_{\boldsymbol{\alpha}_p,n}) \sum_{j=1}^p \beta_m^j \sigma_{m,n}^j.$$

$\because \lambda\sigma_{\boldsymbol{\alpha}_p,n} = \lambda(\lambda + b_n)^{-1} < 1$, $\therefore 2 - \lambda\sigma_{\boldsymbol{\alpha}_p,n} > 1$, $\therefore \sigma_{m,p,n}^N > 0$, $n = 1, \dots, l$, $\therefore \mathbf{D}_{m,p}^N$ is positive definite, and $d_{m,p}^N > 0$. It is obvious that $d_{m,p}^D > 0$. Consequently,

$$d_{m,p}^{t+1} = \frac{d_{m,p}^N}{d_{m,p}^D} > 0,$$

where $m = 1, \dots, M$.

2.2. Proof of Second Conclusion

According to Eq. (2), we have

$$\begin{aligned} d_{m,p}^D &= 2\sum_{j=1}^p \beta_m^j (\mathbf{K}_m^j \boldsymbol{\alpha}_p)^\top (\mathbf{K}_m^j \boldsymbol{\alpha}_p) \\ &= \mathbf{y}_c^\top 2 \sum_{j=1}^p \beta_m^j \mathbf{N}_p \mathbf{D}_p^{-1} (\mathbf{K}_m^j)^2 \mathbf{D}_p^{-1} \mathbf{N}_p \mathbf{y}_c \\ &= \mathbf{y}_c^\top \mathbf{D}_{m,p}^D \mathbf{y}_c \end{aligned}$$

where $\mathbf{D}_{m,p}^D = 2\mathbf{N}_p \mathbf{D}_p^{-1} \sum_{j=1}^p \beta_m^j (\mathbf{K}_m^j)^2 \mathbf{D}_p^{-1}\mathbf{N}_p$, and its n^{th} eigenvalue is

$$\sigma_{m,p,n}^D = 2\sigma_{\boldsymbol{\alpha}_p,n}^2 \sum_{j=1}^p \beta_m^j (\sigma_{m,n}^j)^2.$$

Then, according to Eq. (3),

$$d_{m,p}^{t+1} = \frac{d_{m,p}^N}{d_{m,p}^D} = \frac{\mathbf{y}_c^\top \mathbf{D}_{m,p}^N \mathbf{y}_c}{\mathbf{y}_c^\top \mathbf{D}_{m,p}^D \mathbf{y}_c} = \frac{\mathbf{y}_c^\top \mathbf{U} \mathbf{\Sigma}_{m,p}^N \mathbf{U}^H \mathbf{y}_c}{\mathbf{y}_c^\top \mathbf{U} \mathbf{\Sigma}_{m,p}^D \mathbf{U}^H \mathbf{y}_c}.$$

Let $\mathbf{U}^H \mathbf{y}_c = (y_{u,1}, \dots, y_{u,l})$, $c_n^N = \sum_{j=1}^p \beta_m^j \sigma_{m,n}^j$, and $c_n^D = \sum_{j=1}^p \beta_m^j (\sigma_{m,n}^j)^2$. Then

$$d_{m,p}^{t+1} = \frac{\sum_{n=1}^l y_{u,n}^2 c_n^N \sigma_{\boldsymbol{\alpha}_p,n}(2 - \lambda\sigma_{\boldsymbol{\alpha}_p,n})}{2\sum_{n=1}^l y_{u,n}^2 c_n^D \sigma_{\boldsymbol{\alpha}_p,n}^2}.$$

Let $c_{\max}^N = \max_n c_n^N$, $c_{\min}^N = \min_n c_n^N$, $c_{\max}^D = \max_n c_n^D$, $c_{\min}^D = \min_n c_n^D$, $y_{\max} = \max_n y_{u,n}$, $y_{\min} = \min_n y_{u,n}$,

$$c_l = \frac{y_{\min}^2 c_{\min}^N}{y_{\max}^2 c_{\max}^N}, \quad c_u = \frac{y_{\max}^2 c_{\max}^N}{y_{\min}^2 c_{\min}^N},$$

and

$$\sigma_r = \frac{\sum_{n=1}^l \sigma_{\boldsymbol{\alpha}_p,n}(2 - \lambda\sigma_{\boldsymbol{\alpha}_p,n})}{2\sum_{n=1}^l \sigma_{\boldsymbol{\alpha}_p,n}^2}.$$

Then

$$c_l \cdot \sigma_r < d_{m,p}^{t+1} < c_u \cdot \sigma_r.$$

Furthermore,

$$\begin{aligned} \sigma_r &= \frac{\sum_{n=1}^l (\lambda + b_n)^{-1}(2(\lambda + b_n) - \lambda)(\lambda + b_n)^{-1}}{2\sum_{n=1}^l (\lambda + b_n)^{-2}} \\ &= \frac{\sum_{n=1}^l (\lambda + b_n)^{-2}(\lambda + 2b_n)}{2\sum_{n=1}^l (\lambda + b_n)^{-2}} \\ &= \frac{1}{2}\lambda + \frac{\sum_{n=1}^l b_n(\lambda + b_n)^{-2}}{\sum_{n=1}^l (\lambda + b_n)^{-2}}. \end{aligned}$$

Let $\sigma_{m,\max}^j = \max_n \sigma_{m,n}^j$, $\sigma_{m,\min} = \min_n \sigma_{m,n}^j$,

$$b^{\max} = \frac{\sum_{j=1}^p \sum_{m=1}^M \beta_m^j d_{m,p}^2 (\sigma_{m,\max}^j)^2}{\sum_{j=1}^p \sum_{m=1}, \beta_m^j d_{m,p} \sigma_{m,\min}^j},$$

$$b^{\min} = \frac{\sum_{j=1}^p \sum_{m=1}^M \beta_m^j d_{m,p}^2 (\sigma_{m,\min}^j)^2}{\sum_{j=1}^p \sum_{m=1}^M, \beta_m^j d_{m,p} \sigma_{m,\max}^j}.$$

Then, $b^{\min} \leq b_n \leq b^{\max}$, and

$$\frac{1}{2}\lambda + b^{\min} \leq \sigma_r \leq \frac{1}{2}\lambda + b^{\max},$$

$$\frac{c_l}{2}\lambda + c_l \cdot b^{\min} < d_{m,p}^{t+1} < \frac{c_u}{2}\lambda + c_u \cdot b^{\max}.$$

where $m = 1, \dots, M$.