Appendix: supplementary material

We introduce the onset age individual random variable $t_i = t_0 + \tau_i \sim \mathcal{N}(t_0, \sigma_\tau^2)$ instead of the time shift τ_i. The obtained hierarchical model is equivalent to the one presented in Section 3, with unchanged parameters $\theta = (\overline{y_0}, \overline{c_0}, \overline{m_0}, \overline{A}, t_0, \sigma_\tau^2, \sigma_\xi^2, \sigma_\epsilon^2)$ and equivalent random effects $z = (z_{\text{pop}}, z_1, ..., z_N)$, where $z_{\text{pop}} = (y_0, c_0, m_0, A)$ and $\forall i \in [\![1, N]\!],\ z_i = (t_i, \xi_i, s_i)$. The complete log-likelihood writes:

$$\log q(y, z, \theta) = \sum_{i=1}^{N} \sum_{j=1}^{n_i} \log q(y_{i,j}|z, \theta) + \log q(z_{\text{pop}}|\theta) + \sum_{i=1}^{N} \log q(z_i|\theta) + \log q(\theta) \tag{6}$$

where the densities $q(y_{i,j}|z,\theta)$, $q(z_{\text{pop}}|\theta)$, $q(z_i|\theta)$ and $q(\theta)$ are given, up to an additive constant, by:

$$-2\log q(y_{i,j}|z,\theta) \overset{\text{+cst}}{=} \Lambda \log \sigma_\epsilon^2 + \|y_{i,j} - \eta_{c_0,m_0,t_0,\psi_i(t_{i,j})}(w_i) \circ y_0\|^2 / \sigma_\epsilon^2 \tag{7}$$

$$\begin{aligned} -2\log q(z_{\text{pop}}|\theta) \overset{\text{+cst}}{=}\ & |y_0| \log \sigma_y^2 + \|y_0 - \overline{y_0}\|^2/\sigma_y^2 + |c_0| \log \sigma_c^2 + \|c_0 - \overline{c_0}\|^2/\sigma_c^2 \\ & + |m_0| \log \sigma_m^2 + \|m_0 - \overline{m_0}\|^2/\sigma_m^2 + |A| \log \sigma_A^2 + \|A - \overline{A}\|^2/\sigma_A^2 \end{aligned} \tag{8}$$

$$-2\log q(z_i|\theta) \overset{\text{+cst}}{=} \log \sigma_\tau^2 + (t_i - t_0)^2/\sigma_\tau^2 + \log \sigma_\xi^2 + \xi_i^2/\sigma_\xi^2 + \|s_i\|^2 \tag{9}$$

$$\begin{aligned} -2\log q(\theta) \overset{\text{+cst}}{=}\ & \|\overline{y_0} - \overline{\overline{y_0}}\|^2/\varsigma_y^2 + \|\overline{c_0} - \overline{\overline{c_0}}\|^2/\varsigma_c^2 + \|\overline{m_0} - \overline{\overline{m_0}}\|^2/\varsigma_m^2 + \|\overline{A} - \overline{\overline{A}}\|^2/\varsigma_A^2 \\ & + (t_0 - \overline{\overline{t_0}})^2/\varsigma_t^2 + m_\tau \log \sigma_\tau^2 + m_\tau \sigma_{\tau,0}^2/\sigma_\tau^2 + m_\xi \log \sigma_\xi^2 + m_\xi \sigma_{\xi,0}^2/\sigma_\xi^2 \\ & + m_\epsilon \log \sigma_\epsilon^2 + m_\epsilon \sigma_{\epsilon,0}^2/\sigma_\epsilon^2 \end{aligned} \tag{10}$$

noting Λ the dimension of the space where the residual $\|y_{i,j} - \eta_{c_0,m_0,t_0,\psi_i(t_{i,j})}(w_i) \circ y_0\|^2$ is computed, and $|y_0|$, $|c_0|$, $|m_0|$ and $|A|$ the total dimension of y_0, c_0, m_0 and A respectively. We chose either the current [42] or the varifold [7] norm for the residuals.

Noticing the identity $\eta_{c_0,m_0,t_0,\psi_i(t_{i,j})} = \eta_{c_0,m_0,0,\psi_i(t_{i,j})-t_0}$, the complete log-likelihood can be decomposed into $\log q(y,z,\theta) = \left\langle S(y,z), \Phi(\theta) \right\rangle_{\text{Id}} - \Psi(\theta)$ i.e. the proposed mixed-effects model belongs the curved exponential family. In this setting, the MCMC-SAEM algorithm presented in Section 4 has a proved convergence.

Exhibiting the sufficient statistics $S_1 = y_0$, $S_2 = c_0$, $S_3 = m_0$, $S_4 = A$, $S_5 = \sum_i t_i$, $S_6 = \sum_i t_i^2$, $S_7 = \sum_i \xi_i^2$ and $S_8 = \sum_i \sum_j \|y_{i,j} - \eta_{c_0,m_0,t_0,\psi_i(t_{i,j})}(w_i) \circ y_0\|^2$ (see Section 4.5), the update of the model parameters $\theta \leftarrow \theta^\star$ in the M step of the MCMC-SAEM algorithm can be derived in closed form:

$$\overline{y_0}^\star = \left[\, \varsigma_y^2\, S_1 + \sigma_y^2\, \overline{\overline{y_0}} \,\right] / \left[\, \varsigma_y^2 + \sigma_y^2 \,\right] \qquad t_0^\star = \left[\, \varsigma_t^2\, S_5 + \sigma_\tau^{2\star}\, \overline{\overline{t_0}} \,\right] / \left[\, N \varsigma_t^2 + \sigma_\tau^{2\star} \,\right] \tag{11}$$

$$\overline{c_0}^\star = \left[\, \varsigma_c^2\, S_2 + \sigma_c^2\, \overline{\overline{c_0}} \,\right] / \left[\, \varsigma_c^2 + \sigma_c^2 \,\right] \qquad \sigma_\tau^{2\star} = \left[\, S_6 - 2 t_0^\star S_5 + N {t_0^\star}^2 + m_\tau\, \sigma_{\tau,0}^2 \,\right] / \left[\, N + m_\tau \,\right] \tag{12}$$

$$\overline{m_0}^\star = \left[\, \varsigma_m^2\, S_3 + \sigma_m^2\, \overline{\overline{m_0}} \,\right] / \left[\, \varsigma_m^2 + \sigma_m^2 \,\right] \qquad \sigma_\xi^{2\star} = \left[\, S_7 + m_\xi\, \sigma_{\xi,0}^2 \,\right] / \left[\, N + m_\xi \,\right] \tag{13}$$

$$\overline{A}^\star = \left[\, \varsigma_A^2\, S_4 + \sigma_A^2\, \overline{\overline{A}} \,\right] / \left[\, \varsigma_A^2 + \sigma_A^2 \,\right] \qquad \sigma_\epsilon^{2\star} = \left[\, S_8 + m_\epsilon\, \sigma_{\epsilon,0}^2 \,\right] / \left[\, \Lambda N \langle n_i \rangle_i + m_\epsilon \,\right] \tag{14}$$

The intricate update of the parameters $t_0 \leftarrow t_0^\star$ and $\sigma_\tau^2 \leftarrow \sigma_\tau^{2\star}$ can be solved by iterative replacement.

Similarly to Equation 6, the tempered complete log-likelihood writes:

$$\log q_T(y, z, \theta) = \sum_{i=1}^{N} \sum_{j=1}^{n_i} \log q_T(y_{i,j}|z, \theta) + \log q_T(z_{\text{pop}}|\theta) + \sum_{i=1}^{N} \log q(z_i|\theta) + \log q(\theta) \tag{15}$$

with:

$$-2\log q_T(y_{i,j}|z,\theta) \overset{\text{+cst}}{=} \Lambda \log(T\sigma_\epsilon^2) + \|y_{i,j} - \eta_{c_0,m_0,t_0,\psi_i(t_{i,j})}(w_i) \circ y_0\|^2 / (T\sigma_\epsilon^2) \tag{16}$$

$$\begin{aligned} -2\log q_T(z_{\text{pop}}|\theta) \overset{\text{+cst}}{=}\ & |y_0| \log(T\sigma_y^2) + \|y_0 - \overline{y_0}\|^2/(T\sigma_y^2) + |c_0| \log(T\sigma_c^2) + \|c_0 - \overline{c_0}\|^2/(T\sigma_c^2) \\ & + |m_0| \log(T\sigma_m^2) + \|m_0 - \overline{m_0}\|^2/(T\sigma_m^2) + |A| \log(T\sigma_A^2) + \|A - \overline{A}\|^2/(T\sigma_A^2) \end{aligned} \tag{17}$$

Tempering can therefore be understood as an artificial increase of the variances σ_ϵ^2, σ_y^2, σ_c^2, σ_m^2 and σ_A^2 when computing the associated acceptation ratios in the S-MCMC step of the algorithm. This intuition is well-explained in [25].