Supplemental Material
Estimation of Camera Locations in Highly Corrupted Scenarios: All About that Base, No Shape Trouble

Yunpeng Shi
University of Minnesota
shixx517@umn.edu

Gilad Lerman
University of Minnesota
lerman@umn.edu

1. Proof of the AAB Formula

As is shown in Figure 1, $\Omega(\gamma_1,\gamma_2)$ is exactly the shortest path on the manifold S^2 between $-\gamma_1$ and $-\gamma_2$. Since $I_{AAB}(\gamma_3;\gamma_1,\gamma_2)$ is the length of shortest path between γ_3 and $\Omega(\gamma_1,\gamma_2)$, it can be computed via the following procedure: Let γ_p be the orthogonal projection of γ_3 onto $\mathrm{Span}\{\gamma_1,\gamma_2\}$, then

$$I_{AAB}(\gamma_3;\gamma_1,\gamma_2) = \begin{cases} \measuredangle(\gamma_p,\gamma_3), & \text{if } \frac{\gamma_p}{\|\gamma_p\|} \in \Omega(\gamma_1,\gamma_2); \\ \min(\measuredangle(\gamma_1,\gamma_3),\measuredangle(\gamma_2,\gamma_3)), & \text{otherwise.} \end{cases} \quad (1)$$

By the definition of γ_p it can be expressed as $\lambda_1\gamma_1+\lambda_2\gamma_2$, where $(\gamma_3 - \lambda_1\gamma_1 - \lambda_2\gamma_2)\perp \mathrm{Span}\{\gamma_1, \gamma_2\}$. That is, $\langle\gamma_3-\lambda_1\gamma_1-\lambda_2\gamma_2,\gamma_1\rangle=\langle\gamma_3-\lambda_1\gamma_1-\lambda_2\gamma_2,\gamma_2\rangle=0$. Thus, we obtain the following system of equations for λ_1 and λ_2

$$\lambda_1+z\lambda_2=x \quad (2)$$
$$z\lambda_1+\lambda_2=y, \quad (3)$$

where we recall that $x = \gamma_1^T\gamma_3$, $y = \gamma_2^T\gamma_3$ and $z = \gamma_1^T\gamma_2$. The solution of (2) is given by $\lambda_1 = (x - yz)/(1 - z^2)$, $\lambda_2=(y-xz)/(1-z^2)$. Note that $\gamma_p/\|\gamma_p\|\in\Omega(\gamma_1,\gamma_2)$ if and only if $\lambda_1<0$ and $\lambda_2<0$. That is, when $y<xz$ and $x<yz$,

$$\begin{aligned} I_{AAB}(\gamma_3;\gamma_1,\gamma_2) &= \cos^{-1}(\gamma_p^T\gamma_3) \\ &= \cos^{-1}(\lambda_1\gamma_1^T\gamma_3+\lambda_2\gamma_2^T\gamma_3) \\ &= \cos^{-1}(\lambda_1 x+\lambda_2 y) = \frac{x^2+y^2-2xyz}{1-z^2}. \end{aligned} \quad (4)$$

Otherwise,

$$\begin{aligned} I_{AAB}(\gamma_3;\gamma_1,\gamma_2) &= \min(\measuredangle(\gamma_1,\gamma_3),\measuredangle(\gamma_2,\gamma_3)) \\ &= \cos^{-1}\big(\max\big(\gamma_1^T\gamma_3,\gamma_2^T\gamma_3\big)\big) = \cos^{-1}(\max(x,y)). \end{aligned} \quad (5)$$

This concludes the proof of formula (5).

2. Proof of Lemma 4.1

Proof. Let $l(x_1,x_2)$ denote the shortest path on S^2 connecting the points x_1 and x_2. Let $\boldsymbol{u}_1 = -\boldsymbol{v}_1, \boldsymbol{u} = -\boldsymbol{v}$. Note that $x=\measuredangle(\boldsymbol{v}_2(x),\boldsymbol{u}_1)$ by the definition of $\boldsymbol{v}_2(x)$ and $\boldsymbol{u}_1$.

$$\begin{aligned} f(x) &= \mathbb{E}[\min_{\boldsymbol{y}\in l(\boldsymbol{u}_1,\boldsymbol{u})} d_g(\boldsymbol{v}_2(x),\boldsymbol{y})|\boldsymbol{u}\sim U(S^2)] \\ &= \int \min_{\boldsymbol{y}\in l(\boldsymbol{u}_1,\boldsymbol{u})} d_g(\boldsymbol{v}_2(x),\boldsymbol{y})p(\boldsymbol{u})d\boldsymbol{u} \\ &= \int_{d_g(\boldsymbol{u},\boldsymbol{u}_1)\le x} \min_{\boldsymbol{y}\in l(\boldsymbol{u}_1,\boldsymbol{u})} d_g(\boldsymbol{v}_2(x),\boldsymbol{y})p(\boldsymbol{u})d\boldsymbol{u} \\ &\quad + \int_{d_g(\boldsymbol{u},\boldsymbol{u}_1)> x} \min_{\boldsymbol{y}\in l(\boldsymbol{u}_1,\boldsymbol{u})} d_g(\boldsymbol{v}_2(x),\boldsymbol{y})p(\boldsymbol{u})d\boldsymbol{u} \\ &= \int_{d_g(\boldsymbol{u},\boldsymbol{u}_1)\le x} d_g(\boldsymbol{u},\boldsymbol{u}_1)p(\boldsymbol{u})d\boldsymbol{u} + \int_{d_g(\boldsymbol{u},\boldsymbol{u}_1)> x} x p(\boldsymbol{u})d\boldsymbol{u} \\ &= \frac{1}{4\pi}\left[\int_0^{2\pi}\int_0^x \sin\varphi\cdot\varphi d\varphi d\theta + \int_0^{2\pi}\int_x^{\pi} \sin\varphi\cdot x d\varphi d\theta\right] \\ &= \frac{1}{2}(x+\sin x), \end{aligned} \quad (6)$$

where θ and φ are azimuthal angle and polar angle in spherical coordinate system respectively. □

3. Real Data Experiments

Algorithms	LUD[2]								CLS [3, 4]								ShapeFit [1]							
	None		N-AAB		IR-AAB		1DSfM		None		N-AAB		IR-AAB		1DSfM		None		N-AAB		IR-AAB		1DSfM	
Dataset	$\tilde{e}$	$\hat{e}$	$\tilde{e}$	$\hat{e}$	$\tilde{e}$	$\hat{e}$	$\tilde{e}$	$\hat{e}$	$\tilde{e}$	$\hat{e}$	$\tilde{e}$	$\hat{e}$	$\tilde{e}$	$\hat{e}$	$\tilde{e}$	$\hat{e}$	$\tilde{e}$	$\hat{e}$	$\tilde{e}$	$\hat{e}$	$\tilde{e}$	$\hat{e}$	$\tilde{e}$	$\hat{e}$
Alamo	0.47	1.74	0.43	1.26	**0.38**	**1.06**	0.45	1.84	1.35	2.79	0.52	1.43	**0.4**	**1.2**	0.71	2.2	0.44	1.83	0.42	2.57	**0.39**	**1.54**	0.44	2.04
Madrid Metropolis	1.84	5.94	**1.66**	4.72	**1.66**	**4.47**	1.68	5.46	7.1	11.2	4.16	7.72	**3.68**	**7.06**	4.45	9.08	14	27.3	1.49	9.06	**1.45**	**5.44**	1.47	10.92
Montreal N.D.	0.56	1.22	**0.48**	0.81	0.49	**0.75**	0.56	1.29	0.9	1.79	**0.49**	0.8	0.51	**0.76**	0.68	1.65	0.58	3.25	**0.46**	0.83	**0.46**	**0.78**	0.65	3.66
Notre Dame	0.29	0.85	0.28	0.79	**0.27**	0.81	0.28	**0.78**	1.05	2.12	0.6	1.28	**0.53**	**1.25**	0.73	1.36	0.24	0.96	**0.23**	**0.69**	0.24	0.73	**0.23**	0.7
NYC Library	2.43	6.95	1.84	6.3	**1.62**	5.29	1.86	**5.03**	5.3	8.51	4.33	7.93	**3.88**	**6.93**	4.76	7.34	13.2	14.2	13.1	14.1	**13**	**13.9**	13.3	14.3
Piazza Del Popolo	1.66	5.28	1.42	**5.23**	**1.41**	5.47	1.51	5.34	3.42	6.46	2.2	6.16	**1.84**	**6.08**	2.56	6.14	1.47	6.81	**1.31**	**5.95**	1.35	6.76	1.42	6.75
Piccadilly	2.02	3.87	1.79	3.45	**1.64**	**3.29**	1.85	3.62	3.64	5.42	2.89	4.46	**2.82**	**4.29**	3.37	4.98	13.4	14.2	1.4	4.95	**1.35**	**4.35**	13.4	14.1
Roman Forum	2.21	8.33	1.84	7.86	**1.77**	**7.61**	2.18	8.74	6.2	12.4	3.49	9.3	**4.37**	**8.94**	6.23	12.2	26.7	41	21.4	30.6	**12.1**	**19.5**	15.1	39.3
Tower of London	4.03	17.9	2.74	15.9	**2.67**	**8.85**	3.26	17.5	16	27	5.87	17.5	**2.78**	**9.2**	15.3	26.6	**2.41**	**16.9**	2.49	19.5	2.76	31.4	2.48	17.4
Union Square	7.57	11.7	**7.29**	**11.2**	7.5	11.8	7.97	12.3	8.03	12.5	**7.82**	**12.1**	8.06	12.6	8.6	13.1	12.9	19	**12.7**	19	12.8	19.2	12.8	19
Vienna Cathedral	7.26	13.1	6.41	13.4	6.6	13.9	**5.68**	**11.7**	9.59	13.7	9.4	13.4	9.62	13.9	**7.36**	**11.4**	**28.6**	36.6	28.7	36.7	29.8	**35.9**	28.7	36.6
Yorkminster	2.51	5.26	1.73	**4.32**	**1.7**	4.63	2.05	4.72	5.95	8.72	3.61	**6.1**	**3.44**	6.33	5.87	8.46	19.9	28.4	1.66	15.6	**1.56**	**12.5**	14.7	16.8
Ellis Island	**22**	**22.4**	22.6	23.2	23.8	23.6	22.2	22.8	**20.9**	**22**	22.6	23.3	24.3	23.7	22.5	22.8	26.7	27.7	26.6	**27.5**	**26.5**	27.8	26.7	27.7
Gendermenmarkt	17.5	**38.8**	16.6	38.9	16.7	39.1	**16.5**	38.9	20.7	40.9	18.5	41.3	**17.8**	**40.8**	18.7	**40.8**	32.8	51.6	32.8	51.7	32.9	51.8	32.8	51.6

Table 1. Comparison of naive AAB, IR-AAB and 1DSfM for improving 3 location solvers (LUD, CLS, ShapeFit) using 14 datasets from [5]. Performance Comparison between AAB and 1DSfM on improving current location solvers using datasets from [5]. 10% of edges are removed based on computed statistics. The median and mean distance from the estimated camera locations to the ground truth (provided in [5]) are denoted by $\tilde{e}$ and $\hat{e}$ respectively.

Algorithms	LUD[2]								CLS [3, 4]								ShapeFit [1]							
	None		N-AAB		IR-AAB		1DSfM		None		N-AAB		IR-AAB		1DSfM		None		N-AAB		IR-AAB		1DSfM	
Dataset	$\tilde{e}$	$\hat{e}$	$\tilde{e}$	$\hat{e}$	$\tilde{e}$	$\hat{e}$	$\tilde{e}$	$\hat{e}$	$\tilde{e}$	$\hat{e}$	$\tilde{e}$	$\hat{e}$	$\tilde{e}$	$\hat{e}$	$\tilde{e}$	$\hat{e}$	$\tilde{e}$	$\hat{e}$	$\tilde{e}$	$\hat{e}$	$\tilde{e}$	$\hat{e}$	$\tilde{e}$	$\hat{e}$
Alamo	0.47	1.74	0.37	0.81	**0.35**	**0.59**	0.37	0.93	1.35	2.79	0.36	0.8	**0.35**	**0.6**	0.42	0.98	0.44	1.83	0.36	0.82	**0.35**	**0.72**	0.37	0.91
Madrid Metropolis	1.84	5.94	1.06	2.47	**0.98**	**2.43**	1.39	4.86	7.1	11.2	1.26	2.85	**1.12**	**2.72**	2.17	6.26	14	27.3	**3.03**	**6.78**	4.7	14	21.8	32.3
Montreal N.D.	0.56	1.22	0.38	0.57	**0.37**	**0.56**	NA	NA	0.9	1.79	0.4	0.59	**0.37**	**0.55**	NA	NA	0.58	3.25	0.39	**0.57**	**0.37**	0.58	NA	NA
Notre Dame	0.29	0.85	0.23	0.47	**0.2**	**0.38**	0.27	0.74	1.05	2.12	0.27	0.57	**0.21**	**0.43**	0.66	1.23	**0.24**	**0.96**	0.28	1.66	0.29	1.14	**0.24**	1.32
NYC Library	2.43	6.95	0.81	3.95	**0.61**	**1.37**	NA	NA	5.3	8.51	0.8	2.36	**0.63**	**1.49**	NA	NA	13.3	14.3	1.4	8.16	**0.7**	**2.76**	NA	NA
Piazza Del Popolo	1.66	5.28	0.93	1.55	**0.75**	**1.28**	0.96	2.1	3.42	6.46	0.86	1.42	**0.82**	**1.33**	1.38	2.56	1.48	6.81	0.94	3.56	**0.78**	**1.33**	0.9	1.95
Piccadilly	2.02	3.87	1.24	2.31	**0.9**	**2.04**	2.79	4.62	3.64	5.42	1.21	2.15	**0.97**	**2.03**	2.88	4.54	13.4	14.2	7.04	12	**1.11**	**5.9**	8.99	13
Roman Forum	2.21	8.33	1.47	5.02	**1.15**	**3.66**	4.1	13.9	6.2	12.4	1.88	**5.38**	**1.44**	5.44	8.69	17.2	26.7	41	15.8	31.1	**5.1**	**21.1**	15.5	39.9
Tower of London	4.03	17.9	**2.39**	3.68	2.4	**3.49**	2.78	14.34	16	27	2.45	**4.13**	**2.26**	4.2	9.58	20.4	**2.41**	16.9	2.62	**6.47**	2.6	6.86	5.3	56.5
Union Square	7.57	11.7	5.96	9.84	6.37	11.5	**5.73**	**9.04**	8.03	12.5	**5.91**	**9.15**	10.2	16.5	6.11	9.49	12.9	19	12.7	16.3	13	17.6	**11.7**	**14.1**
Vienna Cathedral	7.26	13.1	3.69	8.88	**2.41**	**8.71**	8.99	17.4	9.59	13.7	8.16	12	**4.65**	**10.8**	9.48	19.1	28.6	36.6	28.9	35.9	**2.12**	**9.11**	24.3	33.8
Yorkminster	2.51	5.26	1.4	3	**1.26**	**2.7**	1.8	3.98	5.95	8.72	2.72	4.46	**1.44**	**2.82**	3.54	5.53	19.9	28.4	3.55	18.4	**1.75**	**4.75**	2.75	6.66
Ellis Island	**22**	**22.4**	22.1	23.2	25.6	25.3	24.3	24.4	**20.9**	22	23	23.5	26.1	25	21	**21.9**	26.7	27.7	26.3	**27.2**	**26.2**	27.3	**26.2**	27.3
Gendermenmarkt	**17.5**	**38.8**	20.9	46.1	34	61.3	17.7	40.2	20.7	**40.9**	22	47.3	33.2	61.7	**19.3**	42.7	32.8	51.7	33.2	55	**32.5**	62.7	33.1	**51.5**

Table 2. Comparison of naive AAB, IR-AAB and 1DSfM for improving 3 location solvers (LUD, CLS, ShapeFit) using 14 datasets from [5]. Performance Comparison between AAB and 1DSfM on improving current location solvers using datasets from [5]. 90% of edges are removed based on computed statistics. The median and mean distance from the estimated camera locations to the ground truth (provided in [5]) are denoted by $\tilde{e}$ and $\hat{e}$ respectively. Even after removing 90% of edges, in most of the cases the maximal parallel rigid subgraph still contains $>50\%$ camera locations. "NA" means that the resulting maximal parallel rigid component had only 16 or less locations, whereas in the rest of cases the maximal parallel rigid component had at least 100 locations.

References

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