A Modular and Unified Framework for Detecting and Localizing Video Anomalies - Supplementary Material

1. Proof of Theorem 1

In [1][page 177], for CUSUM-like algorithms with independent increments, such as MOVAD with independent δ_t, a lower bound on the average false alarm period is given as follows

$$E_\infty[T] \geq e^{\omega_0 h},$$

where h is the detection threshold, and $\omega_0 \geq 0$ is the solution to $E[e^{\omega_0 \delta_t}] = 1$.

To analyze the false alarm period, we need to consider the nominal case. In that case, since there is no anomalous object at each time t, the selection of object with maximum predicted kNN distance in $\tilde{D}_t = \max_i \tilde{D}_{t,i}$ does not necessarily depend on the previous selections due to lack of an anomaly which could correlate the selections. Hence, in the nominal case, it is safe to assume that δ_t is independent over time.

We firstly derive the asymptotic distribution of the frame-level anomaly evidence δ_t in the absence of anomalies. Its cumulative distribution function is given by

$$P(\delta_t \leq y) = P((\max_i \{\tilde{D}_{t,i}\})^m \leq D_\alpha^m + y).$$

It is sufficient to find the probability distribution of $(\max_i \{\tilde{D}_{t,i}\})^m$, the mth power of the maximum kNN distance among objects detected at time t. As discussed above, choosing the object with maximum distance in the absence of anomaly yields independent m-dimensional instances $\{X_t\}$ over time, which form a Poisson point process. The nearest neighbor ($k = 1$) distribution for a Poisson point process is given by

$$P(\max_i \{\tilde{D}_{t,i}\} \leq r) = 1 - \exp(-\Lambda(b(X_t, r)))$$

where $\Lambda(b(X_t, r))$ is the arrival intensity (i.e., Poisson rate measure) in the m-dimensional hypersphere $b(X_t, r)$ centered at X_t with radius r [2]. Asymptotically, for a large number of training instances as $M_2 \to \infty$, under the null (nominal) hypothesis, the output of the neural network converges to the actual nearest neighbor distance, and $\max_i \{\tilde{D}_{t,i}\}$ of X_t takes small values, defining an infinitesimal hyperball with homogeneous intensity $\lambda = 1$ around X_t. Since for a homogeneous Poisson process the intensity is written as $\Lambda(b(X_t, r)) = \lambda |b(X_t, r)|$ [2], where $|b(X_t, r)| = \frac{\pi^{m/2}}{\Gamma(m/2+1)} r^m = v_m r^m$ is the Lebesgue measure (i.e., m-dimensional volume) of the hyperball $b(X_t, r)$, we rewrite the nearest neighbor distribution as

$$P(\max_i \{\tilde{D}_{t,i}\} \leq r) = 1 - \exp\left(-v_m r^m\right),$$

where $v_m = \frac{\pi^{m/2}}{\Gamma(m/2+1)}$ is the constant for the m-dimensional Lebesgue measure.

Now, applying a change of variables we can write the probability density of $(\max_i \{\tilde{D}_{t,i}\})^m$ and δ_t as

$$f_{(\max_i \{\tilde{D}_{t,i}\})^m}(y) = \frac{\partial}{\partial y}\left[1 - \exp\left(-v_m y\right)\right], \quad (1)$$

$$= v_m \exp(-v_m y), \quad (2)$$

$$f_{\delta_t}(y) = v_m \exp(-v_m d_\alpha^m) \exp(-v_m y) \quad (3)$$

Using the probability density derived in (3), $E[e^{\omega_0 \delta_t}] = 1$ can be written as

$$1 = \int_{-D_\alpha^m}^{\phi} e^{\omega_0 y} v_m e^{-v_m D_\alpha^m} e^{-v_m y} dy, \quad (4)$$

$$\frac{e^{v_m D_\alpha^m}}{v_m} = \int_{-D_\alpha^m}^{\phi} e^{(\omega_0 - v_m) y} dy, \quad (5)$$

$$= \left. \frac{e^{(\omega_0 - v_m) y}}{\omega_0 - v_m} \right|_{-D_\alpha^m}^{\phi}, \quad (6)$$

$$= \frac{e^{(\omega_0 - v_m)\phi} - e^{(\omega_0 - v_m)(-D_\alpha^m)}}{\omega_0 - v_m}, \quad (7)$$

where $-D_\alpha^m$ and ϕ are the lower and upper bounds for$\delta_t = (\max_i \{\tilde{D}_{t,i}\})^m - D_\alpha^m$. The upper bound ϕ is obtained from the training set.

As $M_2 \to \infty$, since the mth power of $(1 - \alpha)$th percentile of nearest neighbor distances in training set goes to zero, i.e., $D_\alpha^m \to 0$, we have

$$e^{(\omega_0 - v_m)\phi} = \frac{e^{v_m D_\alpha^m}}{v_m}(\omega_0 - v_m) + 1. \quad (8)$$

Figure 1. The proposed model gives a false alarm in the first and third case and is unable to detect the anomaly in the second case.

We next rearrange the terms to obtain the form of $e^{\phi x} = a_0(x + \theta)$ where $x = \omega_0 - v_m$, $a_0 = \frac{e^{v_m D_\alpha^m}}{v_m}$, and $\theta = \frac{v_m}{e^{v_m D_\alpha^m}}$. The solution for x is given by the Lambert-W function [3] as $x = -\theta - \frac{1}{\phi}\mathcal{W}(-\phi e^{-\phi\theta}/a_0)$, hence

$$\omega_0 = v_m - \theta - \frac{1}{\phi}\mathcal{W}\left(-\phi\theta e^{-\phi\theta}\right). \quad (9)$$

Finally, since the false alarm rate (i.e., frequency) is the inverse of false alarm period $E_\infty[T]$, we have

$$FAR \leq e^{-\omega_0 h},$$

where h is the detection threshold, and ω_0 is given above.

2. Fail Cases

In Fig. 1, we analyze a few cases in which the proposed detector is unable to detect the anomaly or raises a false alarm. In the first case, we see a person standing on the grass, which is considered as anomalous by the proposed detector since there is no similar frame of people standing on the grass in the training data. In the second case, the detector misses a person using a skateboard since the resolution of the video is low and the skateboard is obscured from view. In the last case, the reflection of a person riding a bicycle on the window results in a false alarm.

3. Source Code

We have included the source code along with the extracted features to reproduce the results reported in the paper. However, due to file size constraint, the actual outputs from the optical flow, object detection and pose estimation models are not included. The code for the video anomaly detector is written in Python and has been tested on Ubuntu 18, and the code for the upper bound on the false alarm rate presented in Theorem 1 is written in MATLAB. More details can be found in the readme.md file. The entire code will be made available on GitHub upon the acceptance of the paper.

References

[1] Michèle Basseville and Igor Nikiforov. *Detection of abrupt changes: theory and application*, volume 104. Prentice Hall, Englewood Cliffs, 1993. 1

[2] Sung Nok Chiu, Dietrich Stoyan, Wilfrid S Kendall, and Joseph Mecke. *Stochastic geometry and its applications*. John Wiley & Sons, 2013. 1

[3] Tony C Scott, Greg Fee, and Johannes Grotendorst. Asymptotic series of generalized lambert w function. *ACM Communications in Computer Algebra*, 47(3/4):75–83, 2014. 2